Journal of Ship Research

Parametric Generation, Modeling, and Fairing of Simple Hull Lines With the Use of Nonuniform Rational B-Spline Surfaces

F. L. Pérez,* J. A. Clemente,[†] J. A. Suárez,* and J. M. González*

*Naval Architecture and Marine Engineering School of Madrid, Universidad Politécnica de Madrid, Madrid, Spain [†]Navantia Technical Directorate, Bahía de Cádiz Branch, Cádiz, Spain

> This paper deals with the use of a simple parametric design method applied to simple hull lines, such as sailing ship hulls and round bilge hulls. The described method allows the generation of hull lines that meet hydrodynamic coefficients imposed by the designer, obtaining more flexibility than with normal affine transformations of a parent hull. First, a wire model of the ship stations is made with the use of explicit curves. The method is completed with an automatic surface modeling of the previously generated offsets. The construction of spline curves and their application in the definition of ship lines are reviewed. Approximation of spline curves fitting the data on the stations is made, with special emphasis on the choice of parametrization, which is relevant to increasing the accuracy of the splines. B-spline surface modeling of the hull and the fairing process adapted to maintain certain ship characteristics are described. Some examples of the generation, lofting, and fairing process are presented.

Keywords: ship design; parametric generation; NURBS

1. Introduction

SHIP DESIGN is mainly based on the extensive use of hull databases, and on the designer's experience. In the past it was considered a kind of art, where the experienced designer could imagine the flow lines around the ship hull and the design was a combination of the experience and certain rules that had been successfully used in previous designs. Today, computers and design programs have opened the field of lines design to a wider scope of people. A new field of research called computer-aided ship hull design (CASHD) evolved in the second part of the last century and some computational methods have been reported in recent years (see, e.g., Harries 1998, Kim et al. 1996, Rabien 1996, Tuohy et al. 1996, Kerczek & Stern 1983).

In this paper, parametric generation of hull lines is applied to simple hull forms, such as round bilge lines, normally used in patrol boat hulls or sailing ships. These lines are suitable for this simple method in so far as they have no bow bulbs, integrated skegs, or other complex parts.

The presented method starts with the generation of an offsetbased representation, which meets certain hydrodynamic coefficients imposed by the designer. These coefficients are based on the sectional area curve and on the waterplane, as detailed in section 3. This is done with the use of explicit curves in section 4, and a basic fairing can be made, as explained in section 6. The design method considers both the underwater and the above-water part of the ship, which is described in section 5.

Once the hull offsets are obtained, the nonuniform rational Bspline (NURBS) surface modeling can begin, as described in section 7. A brief background on splines, which are used as approximating curves of the station offsets, is outlined in section 7.1 with special emphasis on the choice of the parametrization to increase the accuracy of the spline fitting. Based on these approximating splines, a NURBS surface is created, as described in section 7.8. The surface fairing is studied thoroughly in section 8. Finally, a sailing cruise ship is generated, creating first the wire model for its station and then a faired NURBS surface according to the method.

Manuscript received at SNAME headquarters March 8, 2006; revised manuscript received June 23, 2006.

For this example, a small prototype was constructed with the surface information generated with this work.

2. Hull shape parametric design

Generally, CASHD can be subdivided into shape representation and shape design. While the former is concerned with accurately reproducing a previously known hull geometry in computer format, the latter mainly deals with the ab initio modeling of a new hull form. A parametric design of the ship hull means that the underwater part of the ship and the above-water part must include some parameters imposed by the designer. For the underwater part, the parameters are well known: block coefficient, LCB, *Awp;* but they can be more complex or not so intuitive, such as certain angles used to define the bow flare, or those related to the definition of bulbous shapes.

Once the main dimensions of the ship have been selected from regression analysis of similar vessels, owner's requirements, or shipyard restrictions, the hull designer chooses, more often than not, to base the design on a suitable hull form of known performance. If this hull is not at hand or the selection is not adequate for the requirements, problems are likely to appear in later stages of the ship design process.

The base design selected is usually one of sufficiently good performance and previously used in other projects. In many cases innovative designs are deemed too risky and discarded. In the case of sailing vessels, hull characteristics must comply with determined ratios depending on the vessel class, thus constraining innovation in the design of hull appendages.

It is possible to adjust the main dimensions and some form coefficients of the base ship by means of parametric transformations, but not all of them in a simple way. In addition, the transformation cannot be successful if the differences between the base ship and the required ship are important.

The presented method is based on the design of the sectional area curve and of the waterplane. The design of the sectional area curve is a very important aspect that has a direct influence on the performance characteristics of the vessel. The basic "rules of thumb" state clearly to get away from hard shoulders and follow certain criteria (see, e.g., Saunders 1957) to locate the center of buoyancy. The sectional area curve and the design waterplane also have a direct influence on the phase between the bow and stern wave trains generated by the hull while the smoothing of shoulders in the sectional area curve affects their amplitude. In addition, the

half entrance angle of the waterline influences both the phase and amplitude of the wave trains, and the design waterplane is directly related to the ship's transverse stability.

After the preliminary body plan has been obtained by any traditional method, it is time to continue through the design process: general arrangement, weight estimation, naval architecture calculations, and so forth. It is not uncommon, however, to perform several modifications to this preliminary hull shape in order to get a balanced design that meets most or all of the requirements. This process of modification and recreation of a new hull shape is, more often than not, quite costly in terms of time and resources (Hamalainen 2002), and it is clearly improved with a parametric generation of the hull.

As is commonly known for the practicing naval architect, if the preliminary project is not adequately carried out, any changes in the design introduced when the building is already in progress are not only time consuming but represent an increase in the budget. The key idea is then to check early in the project development how modifications affect power and speed predictions, stability, general arrangement, and so forth by making use of the appropriate tools (computational fluid dynamics [CFD], etc.) and, in case of conflicting changes, try to come up with a compromise solution. It is in this context that the parametric generation of the hull shape rises as a helpful tool for the designer.

In any case, the importance of the experience of the designer must never be underestimated, in spite of the greater number of numerical tools currently available for the naval architect. The combination of such experience with an adequate tool set will allow the designer to find any critical issues in the hull shape design and to eliminate or compensate for them more quickly and precisely than in the past.

3. Basis of the method

The basis of the proposed method lies in working with the sectional area curve as well as with the waterplane half-breadths curve.

These two curves will be defined mathematically, as is further explained in the following subsection, to be modified later by the designer, for instance, to increase the beam on a certain zone by moving the waterplane center of gravity, but always keeping the curves smooth and without inflections. These new variations will consequently introduce changes in the main hull shape parameters that were selected during the first approximation.

Nomenclature _

α_e = entrance angle of SAC	Bx = maximum half breadth	$p'_{n}, q'_{n} =$ coefficients of the <i>n</i> th station
α_{nc} = angle of the <i>n</i> th station at the deck	G_s = global fairness parameter	(above water)
α_{nf} = angle of the <i>n</i> th station at the	k_n = deadrise at the <i>n</i> th station	$p_n, q_n = \text{coefficients of the } n\text{th station}$
waterplane	LCB = center of buoyancy	(under water)
α_s = trailing angle of SAC	LCF = center of flotation	s(u) = spline curve as a function of u
β_{ae} = half entrance angle of waterplane	LCX = position of the maximum midship	parameter
β_{as} = half trailing angle of waterplane	area	s(u, v) = NURBS surface
∇ = displacement	LCXF = position of the maximum half breadth	SAC = sectional area curve
A_0 = transom wetted area	$L_{kl} = $ local fairness parameter	T = draft
A_n = wetted area of the <i>n</i> th station	LWL = length in waterline	T_n = draft at the <i>n</i> th station
Awp = waterplane area	m + 1 = number of control points of the	u = parameter (vertical)
Ax = maximum midship area	spline	u_i , v_i = knots of the spline or surface
B_0 = transom wetted half breadth	n = degree of the spline	v = parameter (longitudinal)
B_n = half breadth of the <i>n</i> th station at the	$N_i^n = j$ th basis function of a spline with <i>n</i>	$V_i = j$ th control point of a spline
waterplane	degree	$X_j Y_j(X, Y) =$ coordinates of the V_j control point

3.1. Sectional area curve definition

The method described here makes use of a sectional area curve, which is parameterized by the designer using the following magnitudes, commonly used by the naval architect:

- Displacement in cubic meters (∇)
- Longitudinal position of the center of buoyancy (LCB)
- Maximum midship area (*Ax*)
- Longitudinal position of maximum midship area (LCX)
- Waterline length (LWL)
- Entrance angle in the bow (α_e)
- Trailing angle in the stern (α_s)
- Transom wetted area (A_0) .

These parameters can be obtained based on experience, similar vessels, or from literature (see, e.g., chapter 11 of Lamb 2003 or chapter 1 of Lewis 1987).

To generate the curve mathematically, two fourth-degree polynomials are used, one of them for the aftbody (y_1) and the other for the forebody (y_2) , as shown in Fig. 1. All lengths are made dimensionless with LWL and the areas with Ax. In order to obtain the values of the polynomial coefficients, we require a total of 10 conditions for both curves, which are obtained by imposing the following conditions:

Aft body:
$$y_1(x) = A_4 \cdot x^4 + A_3 \cdot x^3 + A_2 \cdot x^2 + A_1 \cdot x + A_0$$
 (1)

Transom area: $y_1(0) = A_0$ = transom area/Ax Stern angle = α_s : $y'_1(0) = tg(\alpha_1) = LWL/Ax \cdot tg(\alpha_s)$ Maximum in LCX: $y_1(xm) = 1$; with $x_m = LCX/LWL$; $y_1(xm) = 0$

Fore body:
$$y_2(x) = B_4 \cdot (x - x_m)^4 + B_3 \cdot (x - x_m)_3 + B_2 \cdot (x - x_m)^2 + B_1 \cdot (x - x_m) + B_0$$
 (2)

Maximum in LCX: $y_2(x_m) = 1$: $y'_2(x_m) = 0$ Bow angle $= \alpha_e$: $y'_2(1) = tg(\alpha_2) = LWL/Ax \cdot tg(\alpha_e)$ Waterline length: $y_2(1) = 0$

Fore body and aft body

Displacement:
$$\int_{0}^{x_{m}} y_{1}(x) dx + \int_{x_{m}}^{1} y_{2}(x) dx = Cx = \frac{\nabla}{A_{x} \cdot LWL}$$

Center of buoyancy:
$$\int_{0}^{x_{m}} x \cdot y_{1}(x) dx + \int_{x_{m}}^{1} x \cdot y_{2}(x) dx = \frac{LCB}{LWL} \cdot Cx$$

As a result of the application of the above conditions, a linear system of 10 unknowns (the polynomial coefficients $A_0 \dots A_4$ and $B_0 \dots B_4$) is obtained and then solved numerically.



Fig. 1 Polynomial definition of sectional area curve

3.2. Waterline definition

To proceed with the definition of the waterline curve, or halfbreadths curve, we follow a procedure analogous to that described to generate the sectional area curve. We can either take an existing waterline from a previous design or generate a new one mathematically from a set of parameters commonly used by the naval architect.

The parameterization chosen for the waterline curve is as follows:

- Waterplane area (*Awp*)
- Longitudinal position of flotation center (LCF)
- Maximum half-breadth (Bx)
- Longitudinal position of maximum half-breadth (LCXF)
- Waterline length (LWL)
- Waterline half entrance angle in bow (β_{ae}) and half trailing angle in stern (β_{as})
- Transom wetted half-breadth (B_0) .

As for the case of the sectional area curve, two fourth-degree polynomials are used to define the waterline curve, one for the aftbody (w_1) and the other for the forebody (w_2) , as shown in Fig. 2. All lengths are made nondimensional with LWL and all breadths with *Bx*. In order to solve for the polynomial coefficients, the following conditions are imposed:

Aft body:
$$w_1(x) = C_4 \cdot x^4 + C_3 \cdot x^3 + C_2 \cdot x^2 + C_1 \cdot x + C_0$$
 (3)

Beam on transom: $w_1(0) = C_0$ = beam on transom/BxHalf trailing angle = β_s : $w'_1(0) = tg(\beta_1) = LWL/Bx \cdot tg(\beta_s)$ Maximum in LCXF: $w_1(x_{mw}) = 1$ with $x_{mw} = LCXF/LWL$; $w'_1(x_{mw}) = 0$

Fore body:
$$w_2(x) = D_4 \cdot (x - x_{mw})^4 + D_3 \cdot (x - x_{mw})^3 + D_2 \cdot (x - x_{mw})^2 + D_1 \cdot (x - x_{mw}) + D_0$$
 (4)

Maximum in LCX: $w_2(x_{mw}) = 1$; $w'_2(x_{mw}) = 0$ Half entrance angle $= \beta_e \cdot w'_2(1) = tg(\beta_2) = LWL/Bx \cdot tg(\beta_e)$ Waterline length: $w_2(1) = 0$

Fore body and aft body:

Waterplane area:
$$\int_{0}^{x_{mw}} w_1(x) \, dx + \int_{x_{mw}}^{1} w_2(x) \, dx = Cwp = \frac{Awp}{B_x \cdot LWL}$$

Center of flotation: $\int_{0}^{x_{mw}} x \cdot w_{1}(x) dx + \int_{x_{mw}}^{1} x \cdot w_{2}(x) dx = \frac{\text{LCF}}{\text{LWL}} \cdot Cwp$



Fig. 2 Polynomial definition of waterline curve

Once again we arrive at a linear system of 10 unknowns that is solved numerically to determine the actual values of the polynomial coefficients.

The parameterization of the waterline curve can be extended by taking into account a transverse metacentric radius (BM) selected by the designer through the existing relationship between the moment of inertia of the waterplane area with respect to the longitudinal axis and BM.

As we have introduced a new parameter, the polynomial defining the aftbody shape is changed to one of fifth degree, $w_1^*(x)$. With this new definition and knowing that GM = KB + BM - KG, if we estimate the height of the center of gravity (*KG*) and the height of the center of buoyancy (*KB*), it is possible to impose a value of *GM*, a magnitude directly related to the stability. Thus, we impose the additional condition BM = GM - KB + KG.

$$BM: \frac{2}{3} \int_0^{x_{mw}} \left(\frac{w_1^*(x)}{2}\right)^3 dx + \frac{2}{3} \int_{x_{mw}}^1 \left(\frac{w_2(x)}{2}\right)^3 dx = \frac{BM \cdot \nabla}{B_x^3 \cdot LWL}$$

We have obtained now a total of 11 unknowns to be solved by numerical means as in the previous cases. It is convenient, once the hull shape is generated as explained in section 0, to check that the calculated *KB* is near the estimated value used to calculate *BM*.

It is beyond the scope of this paper to discuss the mathematical relationships between the different unknowns of the linear systems that define the sectional area curve and the waterline that gives a determinate equation system with valid solutions. For example, a combination of $Xm = 0.1 \cdot Lwl$, LCB = $0.9 \cdot Lwl$, and a block coefficient of 0.1 in the sectional area curve does not give a valid solution. The use of normal design parameters and common sense is the better choice to obtain valid solutions.

4. Hull shape definition

Up to this point we have obtained a sectional area curve and a half-breadths curve in the waterplane that give the ship its main parameters and hydrodynamic coefficients. We need now a third curve to define the longitudinal profile, without appendages. This profile can be directly selected by the designer from other ships or from a preliminary general arrangement if available, and for the sake of simplicity this has been the selected option in this paper. A more complex parameterization of the sectional area curve, waterline, and centerline can be seen in Harries (1998) that also uses the geometric inertia moment of these curves and other geometrical parameters.

This profile has to be smooth, without inflections and with the same waterline length used in the parameterization of the sectional and half-breadths curves. In a later section we will see if the selected curve produces a smooth hull shape and, if not, a procedure to fix the problem as much as possible.

With this set of three curves forming the basis of our hull shape definition, we need to find a suitable formulation to describe the shape of the frames. This formulation must comply with an area imposed by the sectional area curve, with a beam imposed by the half-breadths curve and with the frame feet imposed by the abovementioned profile curve.

For any frame *n*, the following mathematical expression has been selected (Jorde 1997):

$$z = (T - T_n) + k_n \cdot y + p_n \cdot y^{q_n}$$
⁽⁵⁾

In this equation (Fig. 3), z is the ordinate measured from the baseline, y is the distance to the center plane, k_n is the tangent in midship (deadrise angle), T is the draft of the ship, and T_n is the draft of frame n, meaning the distance from the waterplane to the lowest point of the frame of abscissa n, obtained from the profile curve imposed.

The reason for this definition for the shape of the frames lies in the composition of the straight zone, $kn \cdot y$, that defines the shape from the centerplane, and a curved zone, $p_n \cdot y^{qn}$, that leaves the straight zone to match the corresponding beam in the waterplane. This simple expression, however, would not be adequate to define the sections of a bulbous bow or midship sections of full hull forms, such as tankers, that demand the use of spline curves or that have to be divided in different curves.

The values for constants p_n and q_n will be obtained for any frame *n*, with z = T and $y = B_n/2$, where B_n is the value of waterline breadth, obtained from the curve of the waterline half-breadths previously generated.

In addition, the area enclosed by the frame with the flotation and the centerplane (z axis in transverse view) must be equal to $S_n/2$, where S_n is the area of frame n, obtained in this case from the sectional area curve calculated previously. These conditions lead to the following definition of the constants:

$$q_n = \left(T_n - k_n \cdot \frac{B_n}{2}\right) \cdot \frac{B_n}{2} \cdot \left[T_n \cdot \frac{B_n}{2} - \frac{S_n}{2} - \frac{k_n}{2} \cdot \left(\frac{B_n}{2}\right)^2\right]^{-1} - 1 \quad (6)$$
$$p_n = \left(T_n - k_n \cdot \frac{B_n}{2}\right) / \left(\frac{B_n}{2}\right)^{q_n}$$

The sectional area and half-breadths curves are nondimensional, and their values should be multiplied by Ax and Bx, respectively, to obtain the actual values of A_n and B_n for any given frame n.

5. Above-water hull shape

Up to now, only the underwater hull shape has been defined. This is enough to perform numerical optimizations by varying the hull main coefficients, obtaining hull shapes in a relatively straightforward way that is easy to implement in a computer program. For a more realistic approach to the design, it is necessary to fit the complete hull shape with its above-water part. Starting with a hull defined as we have shown in the preceding sections, the designer is faced with several options. All of them require the





definition of a deck line or sheer (Bc_n) , usually a threedimensional curve.

The transverse curves making up the above-water shape lying between the waterline and the deckline can be defined in several ways. The only requirements that have to be imposed are the tangency in the waterplane with the frames of the underwater hull, the location of the ends on the deck line, and that they have to be smooth. It is recommended that the chosen definition allow changes in the frame end angle at the deckline in order to better accommodate changes in the general arrangement.

It is recommended that the designer make use of expressions of the form shown in (7), to make the frames pass through the selected waterline, be tangent to the corresponding underwater frame, and have an end angle on the deckline. According to Fig. 4:

$$z = T + Tg(\alpha_{nf}) \cdot (y - B_n/2) + p'_n \cdot (y - B_n/2)^{q'n}$$
(7)

In this case, the value of α_{nf} is already known from the underwater hull, this being:

$$Tg(\alpha_{nf}) = k_n + p_n \cdot q_n (B_n/2)^{(q_n-1)}$$
 (8)

The values of the constants p'_n and q'_n corresponding to the above-water hull are computed from the conditions already explained. To simplify the notation, we will use $x_1 = B_n/2$, $y_1 = T$, $x_2 = B_{cn}/2$, $y_2 = T_{cn}$.

$$q'_{n} = \frac{(x_{2} - x_{1}) \cdot [Tg(\alpha_{nc}) - Tg(\alpha_{nf})]}{y_{2} - y_{1} - Tg(\alpha_{nf}) \cdot (x_{2} - x_{1})}$$
$$p'_{n} = \frac{y_{2} - Tg(\alpha_{nf}) \cdot (x_{2} - x_{1}) - y_{1}}{(x_{2} - x_{1})^{q'_{n}}}$$
(9)

The above-water hull is extended along the overall length (LOA), that is, over the underwater fore body and aft body. In the frames of these two zones, as there is no underwater hull frame to support the corresponding above-water frame, the value of (8) is unknown and should be estimated, as with α_{nc} , in such a form that the q'_n and p'_n remain smooth along the length, as we will see in section 6. If the vessel has an inclined transom, this has to be taken into account when defining the deck line.



Fig. 4 Definition of above-water hull frames

6. Basic fairing

The mathematical formulation of the frames guarantees their smoothness, but not that of the hull shape as a whole body. Even if smooth curves are used for all defining curves, the hull can show longitudinal irregularities or bumps if all the parameters are not related in a consistent way.

These irregularities can be detected using some magnitudes from the parameterization, or derived from it, that allow the designer to evaluate the "fairness" of the design with respect to some satisfactory criteria. One of these magnitudes is the following coefficient of the sectional area:

$$Cn = \frac{S_n}{B_n \cdot T_n - k_n \cdot (B_n/2)^2}$$
(10)

If we graph this curve, we can see how the shape of the frames changes along the length of the ship. A good design should not show inflections. The greater the value of C_n , but always inside the range [0,1], the fuller the shape of the frames (U form). On the other hand, the lesser the value of C_n , the more the frames take V-like shapes, as in the fore body of a fast ship.

At (10) the designer imposes T_n from the centerplane profile curve selected and k_n from the deadrise distribution chosen, and can achieve a smooth distribution of C_n along the length by varying these two parameters in the problematic zones of the curve, taking into account that the effect of modifying T_n is greater than the one obtained varying k_n . The deadrise can be made constant for all frames or not, as desired.

Other good measures of fairness can be obtained from the variation along the length of the parameters p_n and q_n from (6) as their evolution from one frame to another must be smooth to get a fair hull shape. The effect of these coefficients will be easily understood with the example at the end.

To summarize, in order to delete the possible inflections from any of the three curves above, the designer has to act on T_n and k_n , checking in parallel the centerplane profile curve and, possibly, slightly modifying S_n and B_n , but not forgetting that these changes will alter the initial values of the main coefficients desired for the ship.

Thus, if the changes made on S_n and B_n to smooth out C_n , p_n , and q_n are significant, this means that the initial main coefficients we started with are not consistent, and it is not possible to get a correct hull shape with them. This is an important issue to have in mind when performing a hull shape optimization by altering the main coefficients. Furthermore, the hull fairing will be improved in section 8 with the use of NURBS surface properties.

Using expressions such as (7) for the above-water part, the designer has the advantage of being able to use the q'_n and p'_n curves along the length to check the fairness of the above-water hull. Once again, these curves should be smooth, and for that purpose the designer has to act on the parameter α_{nc} or on the shape of the selected deck line.

7. Modeling the hull with a NURBS surface

Up to this section, a wire model of the ship stations that has the desired form coefficients can be created, but it is possible to define a surface that can rest on the created stations. In the case of these simple hull forms, one surface can model the whole hull. With the use of NURBS surfaces, it is also possible to use a fairing algorithm that will improve the previous fairing of the wire model that was checked with the plot of the p_n , q_n , and *Cs* coefficients. The use of NURBS surfaces enables the creation of the hull mesh, the previous step to CFD calculations, and the naval architecture calculations with computer programs.

In order to define the surface, the offsets obtained with the explicit expressions (5) and (7) will be approximated with spline curves. After this, a lofting surface of these splines can be easily constructed. And finally, once the surface is constructed, the fairing algorithm described in section 8 can be applied.

7.1. Some comments about NURBS

The surfaces that best model the hull of a ship are NURBS surfaces. In order to introduce the notation for the paper, we review briefly how they are produced (see, e.g., Farin 2001, Piegl & Tiller 1997).

A B-spline curve is formed by several pieces of polynomial or rational curves and the whole curve is C^2 (common curvature or second derivatives) at the junctions, in the case of cubic B splines. It is defined by a polygon called a control polygon and by an interpolation algorithm that allows the construction of the curve relating the curve to the control polygon. The interpolation steps are encoded in a family of piecewise polynomial functions $N^n_{j}(u)$ called B-spline functions of degree *n*. The order of the functions is n + 1 and stands for the number of nonnull pieces that the B-spline functions may have. Three is the most usual degree in ship design and the one that better fits the traditional loftsman's splines.

A spline curve is a linear combination of B-spline functions with m + 1 control points as coefficients. So, spline curves are parametric, x = g(u), y = h(u). In the plane Vi = (Xi, Yi), i = 0, ..., m, generate a spline s(u) of degree n:

$$s(u) = \sum_{j=0}^{m} V_j \cdot N_j^n(u) = [X(u), Y(u)] = \sum_{j=0}^{m} [X_j \cdot N_j^n(u), Y_j \cdot N_j^n(u)]$$
(11)

Rational curves may also be defined,

$$s(u) = \frac{\sum_{j=0}^{m} w_{j} \cdot V_{j} \cdot N_{j}^{n}(u)}{\sum_{j=0}^{m} w_{j} \cdot N_{j}^{n}(u)}$$
(12)

by introducing a set of numbers, wj, j = 0, ..., m, called weights. If all of them are one, the polynomial B spline is recovered, since a property of B-spline functions is that their sum is always unity, for all values of the parameter u.

$$\sum_{j=0}^{m} N_j^n(u) = 1$$

The parameter u grows monotonically from one endpoint of the curve to the other. It is usual to take values between zero and one, but this is not necessary. It bears no simple relation to the length of the curve. The price that has to be paid for using parametric coordinates is that the inverse relation, which is the relation that provides u in terms of x or y, is not simple and therefore it is difficult to know the value of u for a given point of the ship offsets.

In addition to the control polygon, a spline curve has also a list of knots, which are the values of the parameter u at the junctions

between pieces. We shall use the word knot to refer either to the junction point or to the value of the parameter at the junction.

The B-spline curve is forced to pass through the first and last vertex of the control polygon, corresponding to knots u = 0 and u = 1. The first and last sides of the control polygon provide the direction of the tangents to the curve at the endpoints. This is achieved by repeating the knots at u = 0 and u = 1 on the spline.

7.2. Calculation of the B-spline functions

A spline curve of degree n is a linear combination of B-spline functions of the same degree. These functions may be constructed recursively from lower to higher degree in terms of the list of knots, starting at u_{-1} . These basis functions can be calculated with the De Boor algorithm of equation (13):

$$N_{j}^{0}(u) = \begin{cases} 1 & u \in (u_{j-1}, u_{j}) \\ 0 & u \notin (u_{j-1}, u_{j}) \end{cases}$$
$$N_{j}^{n}(u) = \frac{u - u_{j-1}}{u_{j+n-1} - u_{j-1}} \cdot N_{j}^{n-1}(u) + \frac{u_{j+n} - u}{u_{j+n} - u_{j}} \cdot N_{j+1}^{n-1}(u)$$
(13)

7.3. Mean square approximation of stations with a cubic spline

As previously mentioned, each station will be given by a list of data points of the ship hull or offsets, formed by p + 1 points $Pi = (xi, yi), i = 0, ..., p, (p \ge n)$, necessarily enclosing the first and last points of the curve, through which the spline will pass. We wish to find a list of m + 1 control points $Vi = (Xi, Yi), i = 0, ..., m, (n \le m \le p)$, which defines the *n*-degree spline of parametric equation s(u), which is closer to the data points, according to equation (11), considering the least square fitting criteria.

We choose cubic splines (n = 3) and not another degree for simplicity of their formulation and for their properties (i.e., possibility of an inflection point in each piece, class C^2). We have also mentioned their similarity with the curves drawn with the traditional design techniques.

7.4. Choosing the list of knots

The spline must pass through the first and last point of the data set. Therefore, the first and last control points must be the endpoints of the original curve and multiplicity three (four, counting the additional knots) will be assigned to the parameter values corresponding to these points. We may choose the remaining knots arbitrarily. They are usually taken equally spaced between 0 and 1 or spaced with constant difference equal to unity:

$$u_{-1} = u_0 = u_1 = u_2 = 3, u_3 = 4, \dots, um = um_{+1} = um_{+2} = um_{+3} = m + 1$$

Once the approximating spline has been defined, we have to choose a function that measures the distance between the spline and the original data. The Euclidean distance between actual and approximating points may be used to define

$$R = \sum_{i=0}^{p} |\mathbf{s}(U_i) - \mathbf{P}_i| = \sum_{i=0}^{p} [(X(U_i) - x_i)^2 + (Y(U_i) - y_i)^2] \quad (14)$$

where Ui, i = 0, ..., p, is the value of the parameter u associated with the point Pi of the data set. This point is related to a point in the spline, s(Ui), and therefore R measures the distance between the spline and the original station.

7.5. Choosing a parameterization

The value of function R will decrease as the spline points approximate to the original data set. For this premise, the choice of parameters Ui, i = 0, ..., p, is determinant. This is called the choice of parameterization. There are many ways of choosing the parameterization. The most usual methods are the uniform parameterization, the parameterization by chord length, and the centripetal parameterization (Farin 2001, Lee 1989).

In a first approximation we will use the centripetal parameterization to obtain the values Ui, $i = 0, \ldots p$, since it usually provides a more precise fitting for a list of points,

$$U_i - U_{i-1} = k \cdot \sqrt{|\mathbf{P}_i - \mathbf{P}_{i-1}|}, \quad i = 1, \dots, p$$
 (15)

where the initial value U_0 and k depend on the origin and width of the interval of definition of the parameter u, in this case, respectively, 3 and m + 1:

$$U_0 = 3, \qquad k = \frac{m-2}{\sqrt{|P_1 - P_0|} + \dots + \sqrt{|P_p - P_{p-1}|}}$$

7.6. Solving the approximation problem

In order to obtain the cubic spline with m + 1 control points, which is closer to the data, we minimize the distance function R, using as free parameters the coordinates of the control points, except for the first and last points, which are already determined. We are left with m - 1 equations for each variable, X, Y:

$$\sum_{j=0}^{m} \left(\sum_{i=0}^{p} N_{k}^{3}(U_{i}) \cdot N_{j}^{3}(U_{i}) \right) \cdot X_{j} = \sum_{i=0}^{p} x_{i} \cdot N_{k}^{3}(U_{i})$$
$$\sum_{j=0}^{m} \left(\sum_{i=0}^{p} N_{k}^{3}(U_{i}) \cdot N_{j}^{3}(U_{i}) \right) \cdot Y_{j} = \sum_{i=0}^{p} y_{i} \cdot N_{k}^{3}(U_{i})$$
(16)

We shall call B the matrix formed by $bij = N^3 j$ (Ui), i = 0, ..., p, j = 0, ..., m:

$$\mathbf{B} = \begin{pmatrix} N_0^3(U_0) & \cdots & N_m^3(U_0) \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ N_0^3(U_p) & \cdots & N_m^3(U_p) \end{pmatrix}$$

The system of equations (16) may be written in matrix form as

$$B^t B X = B^t x \tag{17}$$

where B' denotes the transposed matrix of B, b' ij = bji, and X and x are, respectively, the matrices of coordinates of both the control points and the data points:

$$X = \begin{pmatrix} X_0 & Y_0 \\ \vdots & \vdots \\ \vdots & \vdots \\ X_m & Y_m \end{pmatrix}, x = \begin{pmatrix} x_0 & y_0 \\ \vdots & \vdots \\ \vdots & \vdots \\ x_p & y_p \end{pmatrix}$$

Since the first and last control points are known, because they are equal to the first and last data points, respectively, the first and last columns of this system of equations are moved to the right-hand side. If we write the columns of the matrix B as

$$\mathbf{Bi} = N^3 i(U_0), \ldots, N^3 i(Up)$$

the matrix of the system of equations is provided by their scalar product, C = (cij), $cij = Bi \cdot Bj$, i, j = 1, ..., m - 1:

$$C = \begin{pmatrix} \boldsymbol{B}_1 \cdot \boldsymbol{B}_1 & \cdots & \boldsymbol{B}_1 \cdot \boldsymbol{B}_{m-1} \\ \vdots & \vdots & \vdots \\ \boldsymbol{B}_{m-1} \cdot \boldsymbol{B}_1 & \cdots & \boldsymbol{B}_{m-1} \cdot \boldsymbol{B}_{m-1} \end{pmatrix}$$

This matrix is obviously squared and symmetric and therefore the system

$$C \cdot \begin{pmatrix} X_1 \\ \vdots \\ X_{m-1} \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^{p} x_i \cdot N_1^3(U_i) - \mathbf{B}_1 \cdot \mathbf{B}_0 \cdot X_0 - \mathbf{B}_1 \cdot \mathbf{B}_m \cdot X_m \\ \vdots \\ \sum_{i=0}^{p} x_i \cdot N_{m-1}^3(U_i) - \mathbf{B}_{m-1} \cdot \mathbf{B}_0 \cdot X_0 - \mathbf{B}_{m-1} \cdot \mathbf{B}_m \cdot X_m \end{pmatrix}$$
$$C \cdot \begin{pmatrix} Y_1 \\ \vdots \\ Y_{m-1} \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^{p} y_i \cdot N_1^3(U_i) - \mathbf{B}_1 \cdot \mathbf{B}_0 \cdot Y_0 - \mathbf{B}_1 \cdot \mathbf{B}_m \cdot Y_m \\ \vdots \\ \sum_{i=0}^{p} y_i \cdot N_{m-1}^3(U_i) - \mathbf{B}_{m-1} \cdot \mathbf{B}_0 \cdot Y_0 - \mathbf{B}_{m-1} \cdot \mathbf{B}_m \cdot Y_m \end{pmatrix}$$

has a unique solution. In this case, the Gauss method is used to solve the system of equations of (17). For most ship hull forms, stations are modeled with no more than 15 control points, which can be managed fairly well with the Gauss method.

7.7. Searching for the optimal parameterization

Although the centripetal parameterization usually provides good results, it is desirable to obtain the best spline fitting for the data points of the ship hull with a minimum number of control points. This way the surface fairing process is improved. We shall try to optimize the parameterization by an iterative scheme, using as initial seeds the values Ui, i = 0, ..., p, obtained with the centripetal parameterization.

This is one of the key points of the process, since the quality of the approximation of the station points obviously determines the quality of the hull approximating surface.

Once the approximating spline s(u) is obtained with the values of Ui provided by equation (15), we calculate the line perpendicular to the spline curve from each data point Pi. Such a line cuts the spline at a point Ii so that distance PiIi is the minimum distance between the data point and the spline. Then we take the value U^1 i of the parameter u corresponding to Ii, $Ii = s(U^1 i)$, obtaining a new family of parameters, $U_0^1, \ldots, U^1 p$.

We calculate now the parameterization for the approximating spline $s^1(u)$ with the new parameters, which provides a better precision fitting for the list of points *Pi*. Iterating this process *j* times, we get a family U^j *i* of parameters for which the precision of the associated approximating spline $s^j(u)$ is increased several times compared to the original centripetal parameterization.

In order to calculate the error made in the approximation, we divide the spline in a large number of points and the distance *Pili* is calculated numerically. Figure 5 shows the effect of the parameterization in the fitting of the points of the underwater part of a convex station generated according to section 0 and approximated



Fig. 5 Effect of the parameterization on a convex station. (Top left) Zero iterations, error = 1.11 cm. (Top right) Five iterations, error = 0.96 cm. (Bottom left) Twenty iterations, error = 0.57 cm. (Bottom right) Fifty iterations, error = 0.19 cm

with a spline of four control points, 1.2 m draft, and 2.3 m half breadth. Because of the shape of the stations developed in the presented method, the maximum error values are small. With just five iterations (Fig. 5, top right), the error value is below 1 cm and with 50 iterations (Fig. 5, bottom right), the error value is just a few millimeters.

The effect of the parameterization is increased in the case of stations with changes of curvature. Figure 6 shows the effect of the choice of parameterization in the fitting of the points of a bulbous bow, 6.35 m draft, and 1.7 m half breadth, with a cubic spline of six control points. In Fig. 6 (left) the parameterization is centripetal, the first step of approximation. With five iterations (Fig. 6, middle left), the improvement is hardly perceptible, but the graphs of 20 (Fig. 6, middle right) and 50 iterations (Fig. 6, right) show that the error diminishes with an increase in the number of iterations. This kind of station cannot be defined with the presented method, but it does show better the effect of the parameterization used in this work.

If a certain tolerance is imposed on the approximation, iterations are carried out until the maximal distance *Pili* becomes less than the tolerance value. If the tolerance cannot be attained no matter the number of iterations, the number of control points is to be increased.

Once this process is carried out on the list of points for every station (among them it is also possible to include the stem and stern profiles), we have every station approximated with a cubic spline and also a measure of the approximation error made in each of them. Increasing the number of control points may diminish this error. The tolerance can be taken to be zero by prescribing a number of control points equal to the number of data points, although in this case we are not approximating but interpolating a spline through the data.

Interpolation is just a special case of approximation, but the presented method and the presented examples use approximation instead of interpolation, because this way simpler curves and surfaces can be obtained, facilitating the fairing process. This approximation gives good enough results for the tolerances normally used in ship design.

For the ship hull forms that authors have tested, the parameterization method used in this work does not show convergence problems. The most complex parts of a ship can be bulbous bow forms, as the one in Fig. 6, and these sections are correctly solved with the parameterization process described in this work. Although this kind of section with inflections cannot be generated directly with the presented method and equations (5) and (7), the effect of the parameterization in this kind of section is clearer than in the kind of section with no inflections generated with the method.

7.8. Generation of a spline surface through the stations

The generalization from cubic spline curves to bicubic spline surfaces is almost straightforward. The control polygon is substituted for a control net depending on two indices, *Vij* (*Xij*, *Yij*, *Zij*), as in Fig. 7, that corresponds to the example presented in section 0. Products of B-spline functions in two variables *u* and *v* and two lists of knots $\{u_{-1}, \ldots, um_{+3}\}, \{v_{-1}, \ldots, vn_{+3}\}$ are used:

$$s(u, v) = \sum_{i=0}^{m} \sum_{j=0}^{n} V_{ij} \cdot N_i^3(u) \cdot N_j^3(v)$$

$$u \in [u_2, u_m], v \in [v_2, v_n]$$
 (18)



Fig. 6 Effect of the parameterization on a bulbous bow station. (Left) Zero iterations, error = 6.64 cm. (Middle left) Five iterations, error = 5.24 cm. (Middle right) Twenty iterations, error = 1.84 cm. (Right) Fifty iterations, error = 1.21 cm



Fig. 7 Control point net

For constant u, we obtain cubic spline curves in v with n control points. For constant v, we obtain cubic spline curves in u with m control points. Therefore, we may construct the surface from a two-dimensional net of spline curves. This is especially useful for our purposes, since it allows us to draw a surface from previously designed stations of the vessel.

In practice, this is easy, since it requires just another minimum squares fitting, this time with n + 1 control points for each of the m + 1 longitudinal rows of control points corresponding to the approximating splines for the stations.

These new control polygons constitute the control net of the approximating surface for the initial list of data points. In this case the m + 1 list of points will include the stem and stern profiles.

8. Surface fairing process

Once the surface is fitted with an adequate number of control points, it is usually faired interactively by direct manipulation of the calculated control net, taking into account curvature diagrams of the surface.

The main problem is that generally this fairing process may damage the level of precision acquired during the construction of the approximating surface, since the fairing and fitting process may interfere with each other. To avoid this, we use an automatic fairing method that requires a minimum participation of the user.

It is desirable that the fairing process should be local. If the surface needs to be faired only at a local spot due to the presence of an isolated bump, this should not mean that the whole surface has to be modified, and the hydrodynamic characteristics of the vessel will be maintained.

8.1. Fairing criterion

Among both lists of knots necessary to define the surface, we shall refer to the inner knots as u_3, \ldots, um_{-1} and v_3, \ldots, vn_{-1} , and we assume that they all have a multiplicity of one. The whole set of indices for inner knots (*uk*, v_1) is then I = [(3, 3); (3, 4);; (m-1, n-1)].

If this criterion had to be fulfilled at every knot, it would mean that the spline surface would not be spline but polynomial; that is, it would be a spline of just one piece. Since bicubic spline surfaces are generically C^2 , their third-order derivatives are discontinuous at the knots. That is, we have just excluded the knots corresponding to the edges of the surface.

There are a certain number of fairing algorithms in the literature, but mostly for spline curves and not for surfaces. Methods for spline curves are usually grounded on knot removal procedures (Farin et al. 1987, Kjelander 1983). However, the translation from curves to surfaces is far from straightforward.

We have chosen Hahmann (1998) for its simplicity and its local character, since it involves just nine control points in each iteration, compared with other alternative fairing algorithms (Brunet 1985, Hahmann & Konz 1998, Kjelander et al. 1983). We have modified Hahmann (1998) to maintain certain ship characteristics, as we will explain in the final comments on the fairing process.

Faired surfaces can be generated by means of minimization of certain fairness measures, which can be regarded as approximate strain energy (Nowacki & Reese 1983). A discussion about the selection of proper fairness measures can be found in Moreton and Séquin (1991) and Welch and Witkin (1992). In order to develop a fairing method, we need a sort of quantitative measure of the fairness of the surface.

8.1.1. Local fairness criterion. A C^2 spline surface s(u, v) is fairer in the neighborhood of the inner knot (u_k, v_l) if s(u, v) is locally C^3 at (u_k, v_l) .

Following this criterion, each local fairing iteration consists of reducing the differences between third-order partial derivatives at (uk, v_i) . This means that fairing the whole surface amounts to reducing the value of the sum of third-order discontinuities at all inner knots on the surface.

A spline surface is C^3 at (u, v) if and only if every third-order partial derivative of s(u, v) is continuous at (u, v). Since spline surfaces already have the property of having continuous thirdorder cross partial derivatives, the sum of the differences along both u and v directions provides a reasonable measure of local fairness, according to the fairness criterion.

We may define discontinuity vectors at each knot:

$$\Delta_{uuu}(u_{k}, v_{1}) = \frac{\partial^{3} \mathbf{s}}{\partial u^{3}} (u_{k}^{-}, v_{1}) - \frac{\partial^{3} \mathbf{s}}{\partial u^{3}} (u_{k}^{+}, v_{1}) = \sum_{i=k-4}^{k} \sum_{j=1-3}^{l-1} \alpha_{ij} \cdot \mathbf{V}_{ij}$$
$$\Delta_{vvv}(u_{k}, v_{1}) = \frac{\partial^{3} \mathbf{s}}{\partial v^{3}} (u_{k}, v_{1}^{-}) - \frac{\partial^{3} \mathbf{s}}{\partial v^{3}} (u_{k}, v_{1}^{+}) = \sum_{i=k-3}^{k-1} \sum_{j=1-4}^{1} \beta_{ij} \cdot \mathbf{V}_{ij} \quad (19)$$

where generically the coefficients αij , βij (Table 1 and Table 2) depend on the knot (*uk*, *v_l*). But in the case of equally spaced inner knots, they take the same value for every knot.

Therefore, a local fairness measure Lk_1 at the knot (*uk*, u_l) could be defined as

$$L_{k1} = |\Delta_{uuu}(u_k, v_1)|^2 + |\Delta_{vvv}(u_k, v_1)|^2$$
(20)

In the calculation of Lk_1 , 21 different control points take part.

As a whole, we may assign the whole surface s(u, v) a quantity Gs that may be called a global fairness measure:

$$G_s = \sum_{(k,1)\in I} L_{kl} \tag{21}$$

Table 1 $\alpha i j$ coefficients, $i = k - 4, \ldots, k, j = l - 3, \ldots, l - 1$

αij	k-4	<i>k</i> – 3	k-2	k - 1	k
<i>l</i> – 3	-1/6	2/3	-1	2/3	-1/6
l-2	-2/3	8/3	-4	8/3	-2/3
<i>l</i> – 1	-1/6	2/3	-1	2/3	-1/6

Table 2 $\beta i j$ coefficients, $i = k - 3, \dots, k - 1, j = l - 4, \dots, l$

$m{eta}_{ij}$	l-4	<i>l</i> – 3	l-2	l-1	l
k – 3	-1/6	2/3	-1	2/3	-1/6
k-2	-2/3	8/3	-4	8/3	-2/3
k - 1	-1/6	2/3	-1	2/3	-1/6

8.1.2. Global fairness criterion. A bicubic spline surface s(u, v) is fairer than another s'(u, v) if Gs < Gs'.

The strategy that we follow in order to improve the fairness of the surface involves two steps:

- A local fairing iteration at the knot (*uk*, *v*₁), (*k*, *l*) ∈ *I*, where *Lk*₁ = max (*Lij*), that is, at the least fair knot, according to the local fairness criterion.
- Recalculation of Gs and back to the previous step.

8.2. Local fairing iteration

7

The smoothness of the surface at the knot (uk, u_1) changes from C^2 to C^3 , $Lk_1 = 0$; that is, the third-order derivative discontinuity disappears and the surface is then as smooth as possible at this knot. The local smoothness measure is zero, and therefore,

$$\Delta_{uuu}(u_k, v_1) = 0; \qquad \Delta_{vvv}(u_k, v_1) = 0$$
(22)

This system of equations is compatible but undetermined, since there are more unknowns than equations. We obviously intend to modify the surface minimally; that is, the distance max / s(u, v) - s'(u, v) / s should be minimal. But this condition is highly nonlinear.

In order to linearize the problem, while minimally influencing the surface, we modify the position of just 9 points, instead of 21, keeping the others fixed. That is, we improve the local fairness measure of each knot (*uk*, *v*₁) modifying the position of the control points *Vij*, i = k - 3, k - 2, k - 1, j = l - 3, l - 2, l - 1 that have the greatest effect in the neighborhood of the knot.

In this way, the new position of these nine points will be given by the solution of the system of equations (22) that at the same time causes the smallest deformation of the original surface, that is, the minimum of the vector function F(V' ij):

$$F(\mathbf{V}'_{ij}) = \left(\sum_{i=k-3}^{k-1} \sum_{j=l-3}^{l-1} |v^{x}_{ij} - v'^{x}_{ij}|^{2}, \sum_{i=k-3}^{k-1} \sum_{j=l-3}^{l-1} |v^{y}_{ij} - v'^{y}_{ij}|^{2}, \sum_{i=k-3}^{k-1} \sum_{j=l-3}^{l-1} |v^{z}_{ij} - v'^{z}_{ij}|^{2}\right)$$
(23)

where *Vij* is an original control point and *V'ij* is a modified one. This condition replaces the minimal deformation condition, and

it has the advantage of producing linear Lagrange equations.

In order to solve this problem, we make use of Lagrange multipliers:

$$\Theta(V'_{ij}, \lambda, \mu) = F(V'_{ij}) + \lambda \cdot |\Delta_{uuu}(u_k, v_1)| + \mu \cdot |\Delta_{vvv}(u_k, v_1)|$$
(24)

by imposing that the new function $\Theta(V'ij)$ should meet a minimum. This furnishes the following system of 11 vector equations:

$$\frac{\partial \Theta}{\partial V'_{ij}} = 0 \qquad (9 \text{ equations})$$
$$\frac{\partial \Theta}{\partial \lambda} = 0 \qquad (1 \text{ equation}) \qquad (25)$$
$$\frac{\partial \Theta}{\partial \mu} = 0 \qquad (1 \text{ equation})$$

JOURNAL OF SHIP RESEARCH



Fig. 8 Control point net with few control points

In the case of equally spaced knots, the coefficient matrix of the previous system of equations does not depend on the knot. If the differences uk - uk - 1 and vl - vl - 1 are unity,

Once the inverse of the coefficient matrix A is known, the iterations of the fairing process are easy to compute, since they are just a multiplication of the matrix by the independent term vector. V'_{ii} stand for the new position of the control points V_{ij} after a fairing iteration.

8.3. Final comments on the fairing process

This fairing process will not generally be necessary when the surface has been modeled with a low number of control points, but also with low precision to fit the hull data, since the approximation procedure has already erased small irregularities that may have arisen in the ship offsets. Figure 8 presents two sailing ship hulls generated with the method; the number of control points may be seen, 7×6 , and also the longitudinal distribution of curvature along isoparametric curves, a usual technique for checking the hull fairness.

The problem arises when very high precision is required and a large number of control points are therefore used, even as many as data points. In this case approximation becomes interpolation. In such cases the surface reproduces precisely the ship offsets, but the feared bumps may appear. We may easily detect them plotting the Gaussian curvature of the surface or the curvature of isoparametric curves in any computer-aided design (CAD) application by importing to an IGES file the data that have been obtained in the previous step.

In Fig. 9, a control net of 7×10 points has been used for fitting ship offsets following the method described in this work. A greater number of control points has been used to obtain a better fitting in the bow region. The control net as well as the longitudinal curvature of the isoparametric curves of the surface may be seen.

In these cases we use the fairing algorithm that we have previously described, which is local; that is, it modifies just the surface needed to be smoothed, altering the information of a few control points and leaving the rest unaltered.

However, if we do not want to alter the shape of the ship hull surface at the deck or at the centerline, or the tangent directions on such edges, we must not allow the fairing algorithm to modify the position of every control point. These angles affect ship seakeeping.

In order to avoid this, we fix the position of the last and of the



Fig. 9 Unfaired net



Fig. 10 After seven fairing iterations



Fig. 11 Effect of the fairing algorithm

Table 3 Sailing cruiser

T = 1.26 m	LCB = 9.4 m	$A_0 = 0 \text{ m}^2$	$X_{mw} = 0.4$
$\overline{LWL = 20.3 \text{ m}}$ $LOA = 24$ $\nabla = 49 \text{ m}^{3}$	$X_m = 0.45$ $\alpha_s = 66^{\circ}$ $\alpha_e = 8^{\circ}$	$A_x = 4.4 \text{ m}^2$ $Awp = 72 \text{ m}^2$ $\text{LCF} = 9.2 \text{ m}$	$B_x = 5.1 \text{ m}$ $\beta_s = 21^{\circ}$ $\beta_e = 36^{\circ}$

first two longitudinal rows of control points. We thereby prevent the algorithm from changing them; therefore, the centerline and the bow and stern profiles, as well as the deadrise angle or starting angle of the stations at the centerline, remain unaltered.

In this way we avoid deforming too greatly the original hull surface. In order to achieve this, we restrict just the application of the fairing algorithm to a set I0 of inner knots, subset of the original set I, where the last and the first rows of inner knots have been excluded. That is, the knots that are included in the algorithm and therefore may be faired are (uk, vl) such that k, l are in I' = [(4, 4); (4, 5);; (m - 2, n - 2)].

In Fig. 10 the fairing effects may be seen on the hull of Fig. 9.

The curvature of isoparametric curves has improved if we compare them with the ones that have not been faired.

With the use of a zebra plot, the effect of the fairing process is more visual. In Fig. 11, the hull with seven fairing iterations is shown (left) and compared with the original one without fairing (right).

A final consideration is the required number of iterations of the fairing algorithm. There is no a priori criterion with which to choose it, since it depends on the considered hull. It is not convenient to use a number that is too large so that the original surface does not change too greatly, but it cannot be so low that the surface is not sufficiently faired. A quick view of the curvature of the isoparametric lines and checking the hydrostatic parameters is a good indicator.

9. Application example: recreational cruiser

This section describes an application example. As mentioned, the presented method can be applied for hull forms without bul-



Fig. 12 Prismatic coefficient and position of the center of buoyancy



Fig. 13 Residuary resistance as a function of $L_{wl}/\nabla^{1/3}$.

bous bows or sterns with integrated skegs. Nevertheless, there are many hull forms for which this method can be used for design purposes.

A sailing cruiser of 24 m length is designed. This is the maximum length according to the EC regulation that can be considered for a recreational ship. For this ship, comfort aspects are more important than sailing capabilities because, during an important part of the operational profile, the ship is driven by motor. The ship parameters are shown in Table 3.

The ship is designed to minimize the advance resistance at a speed of 9.5 knots. Resistance is evaluated according the Delft Systematic Series (Gerritsma et al. 1993). Some values, such as CP and LCB, are obtained from Gerritsma et al. (1993) and are shown in Fig. 12.

Displacement was obtained studying the influence of $L_{wl}/\nabla^{1/3}$ in



Fig. 14 Mathematical sectional area curve and waterplane



Fig. 15 Centerline and deck



Fig. 16 Recreational cruiser

Table 4	H١	vdrostatic	parameters
---------	----	------------	------------

	Wire Model	NURBS 0 Fairing	NURBS 7 Fairing
$\overline{\nabla (m^3)}$	49	48.9	48.4
LCB (m)	9.4	9.5	9.6
Awp (m ²)	72	72	71
LCF (m)	9.2	9.1	9.1

the residuary resistance, according to Larsson and Eliasson (1994) (Fig. 13). The value of this ratio was taken to be 5.5. Greater values will produce a faster ship but will reduce the comfort and seakeeping properties.

The generated waterline should consider the internal volume distribution, based on an initial volume study of the internal arrangement of the design. The same consideration can be made for the deck parameters. Maximum breadth is calculated considering sail area and stability parameters. This way, *Awp*, *Bx*, and LCF are obtained.

The sectional area curve and waterplane area obtained with these parameters and equations (1) to (4) can be seen in Fig. 14. Centerline and deck profiles have been obtained to enclose the internal volumes of the general arrangement and can be seen in Fig. 15. The stations calculated are depicted in Fig. 16 (left). The variation of the coefficients C_n , p_n , and q_n is plotted in Fig. 17.

The creation of the sailing cruiser's NURBS surface is shown in Figs. 9 and 10. The final result is depicted in Fig. 16 (right). As mentioned, the NURBS modeling and the fairing algorithm will change the shape of the surface, and the effect on some hydrostatic parameters can be seen in Table 4. Precision can be improved by increasing the number of control points of the surface net, but this will diminish the effect of the fairing algorithm. As in other aspects of ship design, the best solution is a compromise between precision and fairing based on the designer's experience.

A small model of this hull was constructed using the approximating surface from offsets shown in Fig. 16 (left), modeled with the presented method in section 7, and faired with the method described in section 8. The final result is shown in Fig. 18.

10. Conclusions

The method described in this article is able to impose on a hull shape a series of main coefficients commonly used by the naval architect and of great interest for the design. This method allows taking control of the fairing of the hull shape from a certain set of parameters built into its definition. This basic fairing improves the results of the NURBS fairing algorithm.

The hull shape thus generated can be exported to naval architecture programs or can be used for CFD evaluation and be the initial case of an optimization process. To ease this process, an automatic lofting with NURBS surfaces of the generated wire model has been presented.

The lofting method relies on an accurate fitting of data points of the stations with an original selection of the parameterization, increasing as much as possible the precision of the approximating cubic spline curves. The origin of the method is the traditional design with physical splines.

To this aim, an iterative algorithm has been devised for obtaining the values of the optimized parameterization for the spline curves that approximate the hull data points. Other parts of the process, the approximation scheme, or the fairing algorithm are standard, although the fairing has been adapted in order to maintain certain ship characteristics.

The proposed method provides accurate results for hull forms generated with it, although the small changes of the hull shape because of fairing will slightly affect the hydrodynamic parameters, and the best solution should be a compromise between precision and fairing based on the designer's experience.

The NURBS fairing method, however, may be completed. For



Fig. 17 Variation of Q, P, and Cs along ship length (underwater part) for the cruiser



Fig. 18 Constructed model of the recreational cruiser

instance, other fairing algorithms may be tried, such as directional methods (Feng 1997), for elongated hull shapes. This could be another line of research for further progress.

Acknowledgments

The authors wish to acknowledge the assistance and comments of José Ramón García and Leonardo Fernández.

References

- BRUNET, P. 1985 Increasing the smoothness of bicubic spline surfaces, Computer Aided Geometric Design, 2, 157–164.
- FARIN, G. 2001 Curves and Surfaces for CAGD: A Practical Guide, 5th ed., Morgan Kaufmann, San Francisco, CA.
- FARIN, G., REIN, G., SAPIDIS, N., AND WORSEY, A. J. 1987 Fairing cubic B-spline curves, *Computer Aided Geometric Design*, 4, 91–104.
- FENG, J. 1997 Directional surface fairing of an elongated shape, *Proceedings*, FAIRSHAPE Conference, September 14–17, Kleinmachnow, Germany, 163–178.
- HAHMANN, S. 1998 Shape improvement of surfaces, Computing Supplement, 13, 35–152.
- HAHMANN, S., AND KONZ, S. 1998 Knot-removal surface fairing using search strategies, *Computer Aided Geometric Design*, 30, 131–138.
- HAMALAINEN, R. 2002 A most effective way to optimise hull form by using parametric hull definition and CFD, *Proceedings*, IMAM 2002, Crete, Greece.
- HARRIES, S. 1998 Parametric Design and Hydrodynamic Optimization of Ship Hull Forms, Ph.D. thesis, Technical University of Berlin, Berlin, Germany.
- GERRITSMA, J., KEUNING, J. A., AND VERSLUIS, A. 1993 Sailing yacht per-

formance in calm water and in waves, *Proceedings*, 11th Chesapeake Sailing Yacht Symposium, January 29–30, Annapolis, MD.

- JORDE, J. H. 1997 Mathematics of a body plan, *The Naval Architect*, January, 38–42.
- KERCZEK, C., AND STERN, F. 1983 The representation of ship hulls by conformal mapping functions, JOURNAL OF SHIP RESEARCH, 27, 3.
- KIM, S. Y., KIM, H. C., AND LEE, Y. S. 1996 Initial hull design using fuzzy modeling, *Ship Technology Research*, 43, 175–181.
- KJELLANDER, J. A. 1983 Smoothing of bicubic parametric surfaces, Computer Aided Design, 15, 289–293.
- LAMB, T. 2003 Ship Design and Construction, Society of Naval Architects and Marine Engineers, Jersey City, NJ, Chapter 11.
- LARSSON, L., AND ELIASSON, R. 1994 Principles of Yacht Design, Adlard Coles Nautical, London.
- LEE, E. 1989 Choosing nodes in parametric curve interpolation, *Computer Aided Design*, **21**, 363–370.
- LEWIS, E. V. 1987 *Principles of Naval Architecture*, vol. 1, Society of Naval Architects and Marine Engineers, Jersey City, NJ.
- MORETON, H., and SÉQUIN, C. 1991 Surface design with minimum energy networks, *Proceedings*, Symposium on Solid Modelling Foundations and CAD/CAM Applications, June 5–7, Austin, TX, 291–301.
- NOWACKI, H., AND REESE, R. 1983 Design and fairing of ship surfaces, in Barnhill, R.E., and Boehm, W., editors, *Surfaces in CAGD*, North-Holland, Amsterdam, 121–134.
- PIEGL, L., AND TILLER, W. 1997 The NURBS Book, 2nd ed., Springer Verlag, Berlin.
- RABIEN, U. 1996 Ship geometry modeling, *Ship Technology Research*, 43, 115–124.
- SAUNDERS, H. E. 1957 Hydrodynamics in Ship Design, 1st ed., Society of Naval Architects and Marine Engineers, Chapter 20.
- TUOHY, S., LATORRE, R., AND MUNCHMEYER, F. 1996 Developments in surface fairing procedures, *International Shipbuilding Progress*, 43, 281–314.
- WELCH, W., AND WITKIN, A. 1992 Variational surface modelling, Proceedings, SIGGRAPH, Chicago, IL, 26–31.