EXPERIMENTAL EVALUATION OF THE ULTIMATE BENDING MOMENT OF A THIN BOX GIRDER

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ABSTRACT
The results of a four points bending test on a box girder are presented. The experiment is part of series of tests with similar configuration but different thickness and span between frames. The present work refers to the slenderest plate box girder with a plate’s thickness of 2 mm but with a short span between frames. The experiment includes initial loading cycles allowing for residual stresses relief. The moment curvature relationship is established for a large range of curvature. The ultimate bending moment of the box is evaluated and compared with the first yield moment and the plastic moment allowing the evaluation of the efficiency of the structure. The post buckling behavior and collapse mode are characterized. Comparison of the experiment with a progressive collapse method is made taking into consideration the effect of residual stresses on envelop of the moment curvature curve of the structure.

INTRODUCTION
The ultimate bending moment that the transverse section of a ship or a floating production and offloading platform (FPSO) can resist under overall longitudinal bending, is one of the main criteria for design of these structures. The move of the industry towards more accurate predictions of the strength of these structures in overall bending to resist still water and wave induced loads requires accurate and expedite methods to assess the ultimate strength.

Caldwell [1] was the first who addressed the plastic collapse of a ship hull under overall bending although he did not allow for buckling of plate elements as pointed out by Faulkner [2]. The first attempt to incorporate the influence of the buckling collapse of some elements of the cross section was due to Smith [3], who used load shortening curves of individual plate elements to calculate their contribution to the ultimate bending moment of the structure. Other methods based on this general idea were developed including the earlier ones of Billingsley [4], Adamchak [5] and Gordo et al. [6].

This type of progressive collapse methods usually consider that the structural behavior of the hull girder under bending moment may be represented by the summation of the individual contributions of each longitudinal stiffened plate that is part of the cross section. The two main assumptions are that the net longitudinal force in a cross section is zero and the bending moment resulting from the external loads is equal to the first moment of the forces developed in the cross section due to the curvature of the hull girder. The first assumption requires the reevaluation of the location of the neutral axis at every incremental curvature change because of the elasto-plastic nature of the load shortening relationship for each stiffened plate element.

The ultimate moment supported by the hull is achieved after some of the elements have already collapsed, so the knowledge of shedding pattern after buckling of such elements is of great importance. Usually these methods ignore the interaction between adjacent elements thus the calculated ultimate moment may be considered as an upper limit for the maximum bending moment. The main problem associated with such structures is the nonlinear behavior of the components under compression, which is a source of uncertainty on the determination of the ultimate carrying capacity of the structure, especially in a situation of overall bending where some parts are in compression and others in tension.

Because of their nature, these methods require validation by experimental results. However the number of test results available in the open literature is still limited. Two box girders representative of bridges were tested by Dowling et al. [7] and Nishihara [8] tested seven models of scaled and simplified ship cross sections. An experiment on 1/3 scale model of a frigate was performed by Dow [9], but this was a transversely framed ship which is not representative of most present day structures.
The predictions of the method of Gordo et al. [6] reproduced well these tests results [10], but due to the limited extend of geometries involved it was decided to initiate a series of tests that would consider other geometries, covering a wider range of the different parameters that affect the ultimate carrying capacity of such structures under bending.

In this work the results of a test on a box girder representing the mid-ship region of a ship type structure are presented and analyzed. The specimen is subjected to pure bending leading to a mode of collapse in which the upper flange failed under compressive loads.

This result belongs to a series of 5 tests on mild steel box girders [11] where different plate’s thickness and frame’s spacing where used for the same transverse configuration of the box.

After that the authors have performed tests on high and very high tensile steel box girders [12, 13] that allow understanding the influence of the material properties on the ultimate bending moment supported by this type of structure.

**EXPERIMENTAL DETAILS**

**Geometry of the specimen**

The specimen is a one-meter long box girder supported by two blocks of two meters with much higher rigidity than the first. The liaison between them is bolted in order to allow the use of the supports in the future to test other models.

The four points bending test is sketched in Fig. 1 and it allows obtaining pure constant bending throughout the whole specimen.

The central block represents the cross section of a rectangular box girder and has the major dimensions of 800mm wide and 600mm of depth. The span between the two frames of the specimen is 800mm allowing 100mm in each side for redistribution of stresses.

The horizontal panels, top and bottom, have three longitudinal stiffeners equally spaced (200mm) and the lateral webs have only one stiffener each, as presented in Fig. 2. The plate is 2mm thick and the stiffeners are bars with a thickness of 3mm and 30mm of depth. This specimen was designated M2-200.

![Fig. 1 Layout of the experiment and real structure.](image)

![Fig. 2 Cross section (top) and stiffeners arrangement (bottom).](image)

**Material Properties**

In the design phase of the specimen it was considered that the material to be used would be mild steel with a yield stress (σy) of 240 MPa and an elasticity modulus (E) of 210 GPa. Normal ship building steel shows a marked yield followed by a yielding plateau until 8 to 10 times the yield strain. The hardening is not very marked from this point to the ultimate strain which is normally above 20% of the initial length.

<table>
<thead>
<tr>
<th>Nominal thickness (mm)</th>
<th>Nominal Dimensions (mm)</th>
<th>Yield stress (MPa)</th>
<th>Maximum stress (MPa)</th>
<th>Maximum Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.96x12.5</td>
<td>190</td>
<td>280</td>
<td>39.2</td>
</tr>
<tr>
<td>2</td>
<td>1.96x12.4</td>
<td>170</td>
<td>270</td>
<td>42.8</td>
</tr>
<tr>
<td>2</td>
<td>1.96x12.4</td>
<td>170</td>
<td>270</td>
<td>48.8</td>
</tr>
<tr>
<td>3</td>
<td>3.0x12.6</td>
<td>170</td>
<td>280</td>
<td>49.7</td>
</tr>
<tr>
<td>3</td>
<td>3.0x12.6</td>
<td>200</td>
<td>300</td>
<td>47.1</td>
</tr>
<tr>
<td>3</td>
<td>3.0x12.6</td>
<td>180</td>
<td>280</td>
<td>49.0</td>
</tr>
<tr>
<td>2</td>
<td>Average</td>
<td>177</td>
<td>273</td>
<td>43.6</td>
</tr>
<tr>
<td>3</td>
<td>Average</td>
<td>183</td>
<td>287</td>
<td>48.6</td>
</tr>
</tbody>
</table>

Tension tests were performed in order to obtain realistic values for the material properties and the results obtained show some different values relatively to the initial assumptions. Fig. 3 and 4 are the output of those tension tests and Table 1 summarizes the main characteristics obtained with mean values of the
yielding stress of 177 MPa (2mm) and 183 MPa (3mm) and very high ductility.

As may be observed, 2mm plate does not show a marked yield point or yielding plateau and the yield stress is very low. The stiffeners (3mm plate) have a higher yield stress and a marked yield point, followed by a short plateau at constant stress that extends to 2.5 times yield strain.

Fig. 3 Tensile tests of 2mm thick plate specimens

Fig. 4 Typical tensile test of 3mm thick plate specimen

EXPERIMENTAL RESULTS

The experiment was conducted in several cycles of loading followed by total discharges. This procedure was adopted due to the existence of residual stresses in the specimen. During the initial loading cycles the residual stresses in the panel under tension was reduced to very low values. Thus its effect on the early stage of loading is removed and the initial structural modulus (EI) may be obtained from the experiment and compared with the calculated value.

Fig. 5 shows the load vs. global vertical displacement relationship obtained in the four cycles of loading.

Fig. 5 Load-vertical displacement curves for 4 cycles of loading

The first two cycles reached the same maximum load of 70 KN, the third cycle imposed a vertical displacement that goes through collapse and beyond and a last cycle after the collapse of the structure to analyze the elastic and plastic properties of the damage structure.

According to the usual model of residual stresses, the energy dissipates by plastic flow near the longitudinal stiffeners at the bottom, which is in tension. According to the same model it is not expected to have any relief of stresses at the top, which is in compression, at least for low load levels [14]. The stress relief results in an increased residual deformation after each cycle. This box girder is the slenderest of this series of tests. The series varied essentially the slenderness of the plate despite having taken care to ensure that the slenderness of the column was kept at acceptable values and in accordance with the normal practices of construction [11, 15].

Fig. 6 Load-displacement relationship for initial cycles

The results obtained in the test of the box M2-200 show a relationship between the applied force and displacement imposed quite soft, with an absolute maximum at 13 mm of vertical displacement which corresponds to the maximum applied force of 173 KN.

The sequence of charge and discharge in the two first cycles, Fig. 6, shows that the only consequences of these cycles are the relief of residual stresses and dissipation of energy by structural hysteresis. In fact the discharge of the 2nd cycle overrides the
discharge of the 1st indicating that no further plastic had occurred. The energy absorbed in the 1st cycle was 39.2 J which includes dissipation of energy by plasticity in tensile strips near the stiffeners of the bottom panel and Bauschinger’s energy due to structural hysteresis while the 2nd cycle only accounts for this last component and the dissipation energy measured was 9.0 J. So the energy absorbed by residual stresses relief after the initial cycles was 30.2 J. From Fig. 6 it seems that the process of residual stresses relief initiates at 60kN indicating that some plastic deformations had already occurred during the setup of the test and the 30.2 J corresponds to the energy dissipated from 60 to 70 kN.

**Relationship between the moment and the curvature**
The bending moment is calculated directly from the product of the applied force and the distance from the lateral support to the nearest point of loading; the curvature was calculated by gauges measurements in two similar auxiliary devices located in each side of the box. The moment curvature curve is presented in Fig. 7.

![Moment-curvature relationship for complete test](image1)

**First and second load cycles**
In the first two cycles of load a maximum vertical displacement of 5mm was applied generating a bending moment of 71 kNm, which proved to correspond to 42% of the maximum load capacity of the box to pure bending. The results of these two cycles are presented in Fig. 8.

Three important aspects of the results deserve special mention:

1. The transverse rotation of the model at low level of load is proportional to the difference of curvatures measures by the two opposite gauges;
2. the verticality of the curve when the mean curvature, M(C), is between 20 and 60 kNm;
3. the increase of the curvature at the start of the discharge of the load.
4. Regarding the relative rotation between the two transverse faces that can be called more appropriately twisting of the beam, it grows at an early stage, thereby reducing substantially from 50KNm and reaching values important in the discharge phase. Given its almost complete disappearance at greatest loads, it is reasonable to conclude that this rotation is not due to load imbalances but to the internal rearrangements of the geometry of the structure imperfections during the initial charge due to relief of residual stresses.

![M-C curves for first two cycles of loading](image2)

**Collapse load cycle**
The collapse cycle has two different regions in the pre collapse field as shown in Fig. 9: in the first part until the maximum load of previous cycles, the structural behavior is similar in all respects to the second cycle.

![M-C curves for collapse and damaged cycles of loading](image3)

This is result of not having developed any further permanent deformation in the second cycle since it repeated the maximum load of the first cycle, and so, all deformations were reversible in elastic domain; the second part involves the remaining zone until the collapse, where there is a progressive reduction in the
stiffness of the section until collapse. The loss of rigidity is due either to the relief of residual stresses or elasto-plastic deformation associated with the loss of effectiveness of the plates with the development of large out of the plane deformations of the panels.

The loss of load capacity after the collapse in the box is quite smooth which is associated with large energy absorption.

The deformation of the panel in compression during collapse is illustrated in Fig. 10. It is visible the ruin of one of the ribs in the region of the absence of welding and it can be identify perfectly that the global collapse is due to the instability of the plate elements. The permanent deformations induced by the collapse of the top panel on the side panel are also quite high.

The dominant wavelength after the collapse of the plate is approximately 250mm, which corresponds to \( m=6.4 \) and it is much higher than the aspect ratio of the plate (\( \alpha=4 \)). It therefore confirms the results obtained for the ultimate plate strength, which concluded that the minimum resistance is obtained for failure modes higher than the critical mode (\( m > \alpha \)) when there is any degree of restrain on the movement of the lateral edges of the plate elements due to action of the transverse frames [16]. The ratio of the half-wave length at collapse to the associated plate width is 0.625.

**Load cycle of damaged structure**

In Fig. 9 the response of the damaged structure is also represented and one can identify the following characteristics of the curve:

1. The ascending part of the curve in the initial zone has the same slope of the beginning of unloading on the previous cycle.
2. The second part of the ascending curve has a slope considerably lower than former and it is of the same order of magnitude as the structural modulus of the final stage of unloading in previous cycle;
3. The curve after collapse under elasto-plastic domain is resumed approximately at the same point where it started the discharge in the previous cycle.
4. The new rate of load reduction in post collapse is lower than the one in the previous cycle.
5. The discharge in this cycle guarantees the maintenance of the two regions mentioned above but the structural modulus have lower stiffness as a result of a greater deformation of the geometry of the box.

Fig. 11 shows the final deformations on the top panel. It is evident that the collapse is due to plate induced failure, followed by the ruin of the ribs which is facilitated by the intermittent welding of them to the plating. In fact the main deformations on the plating cross the ribs in places where there is no welding.
**Structural Modulus**

Fig. 12 presents the variation of the structural modulus with the curvature on the collapse cycle for bending moment above 70 kNm. From the moment that point is exceeded, the permanent deformation stabilizes the deformed geometry, leveling off also the value of tangent modulus despite a decreasing trend until collapse.

The structural tangent modulus in the post-collapse is practically constant, varying its value around -10MNm². During the removal of the load it can be identified the two types of discharge already mentioned previously as shown in Fig. 13.

As may be observed in Table 3 the ultimate strength of the plating and of the stiffener with associated plating in compression is always lower than the ratio between the ultimate bending moment and the first yield bending moment (0.708). It means that overall structure in bending tends to compensate the reduction in the ultimate strength of the components in compression due to buckling.

**Table 2 Geometrical characteristics and strength of box girder**

<table>
<thead>
<tr>
<th>Item</th>
<th>Designation</th>
<th>Value</th>
<th>% of YM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Area</td>
<td>A (mm²)</td>
<td>6320</td>
<td></td>
</tr>
<tr>
<td>2nd Moment of Area</td>
<td>I (dm⁴)</td>
<td>4,127</td>
<td></td>
</tr>
<tr>
<td>Yield Moment</td>
<td>YM (kN.m)</td>
<td>243.5</td>
<td>100.0</td>
</tr>
<tr>
<td>Plastic Moment</td>
<td>PM (kN.m)</td>
<td>261.9</td>
<td>107.6</td>
</tr>
<tr>
<td>Ultimate Moment</td>
<td>UM (kN.m)</td>
<td>172.8</td>
<td>70.8</td>
</tr>
</tbody>
</table>

The low value of ultimate bending moment is result of high slenderness of the plating and of the panel itself. The plate’s slenderness, given by \( \beta = \frac{b}{t} \sqrt{\frac{\sigma_s}{E}} \) \( \beta = \frac{b}{t} \sqrt{\frac{\sigma_s}{E}} \), is 2.9 in upper range of normal ship plating, and the column’s slenderness, given by \( \lambda = \frac{1}{(\pi r)} \sqrt{\frac{\sigma_s}{E}} \lambda = \frac{1}{(\pi r)} \sqrt{\frac{\sigma_s}{E}} \), is 1.03. For this box one has \( b=200\text{mm}, t=2\text{mm}, l=800\text{mm and} r=7.2\text{mm} \).

**Table 3 Geometrical characteristics and strength of stiffened panel**

<table>
<thead>
<tr>
<th>Item</th>
<th>Designation</th>
<th>Value</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plating area</td>
<td>Ap (mm²)</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>Stiffener’s area</td>
<td>As (mm²)</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Cross area</td>
<td>A (mm²)</td>
<td>490</td>
<td></td>
</tr>
<tr>
<td>Effective cross area</td>
<td>Ape (mm²)</td>
<td>318</td>
<td></td>
</tr>
<tr>
<td>Gyration radii*</td>
<td>r (mm)</td>
<td>7.21</td>
<td></td>
</tr>
<tr>
<td>Yield stress</td>
<td>( \sigma_u ) (MPa)</td>
<td>177</td>
<td>1.000</td>
</tr>
<tr>
<td>Faulkner’s plate stress</td>
<td>( \sigma_{fp} ) (MPa)</td>
<td>101</td>
<td>0.570</td>
</tr>
<tr>
<td>Euler’s column stress*</td>
<td>( \sigma_c ) (MPa)</td>
<td>109</td>
<td>0.617</td>
</tr>
<tr>
<td>Ultimate panel stress*</td>
<td>( \sigma_u ) (MPa)</td>
<td>85</td>
<td>0.478</td>
</tr>
<tr>
<td>Ultimate panel stress**</td>
<td>( \sigma_a ) (MPa)</td>
<td>114</td>
<td>0.644</td>
</tr>
</tbody>
</table>

* Calculated with total area of the associate plate[17].
** Calculated with effective area of the associate plate.

**STRUCTURAL ANALYSIS**

The geometry of the box and the material properties of the steel indicate that the box is very thin and may be representative of the structural behavior of thin ship’s structures.

Table 2 summarizes the relevant structural characteristics of the box and it indicates that the experimental value of the ultimate bending moment (UM) is far below the first yield bending moment (YM), only 71%. The fully plastic bending moment (PM) is only 8% bigger than the first yield moment which is a typical reference value in this type of structures.

Fig. 14 compares the experimental moment-curvature relationship with the predictions from of HullCol software[6] considering residual stresses in the compressive range[18] and the plasticity effect on the stiffened panel in tension[14].

As may be observed the prediction using a residual stress level of \( \eta=3 \) fits particularly well the experimental results, both in the pre and post-collapse region and the ultimate bending moment itself. This tensile width of residual stresses equal to \( 3\% \) of the pre

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compares the predictions with the experimental result and Fig. 15 presents them graphically.

![Comparison between experiment and predictions from progressive collapse methods](image)

Fig. 15 Comparison between experiment and predictions from progressive collapse methods

### Table 4 Ultimate bending moment and ultimate curvature

<table>
<thead>
<tr>
<th>$\sigma_r/\sigma_o$</th>
<th>UM (kN.m)</th>
<th>UC (1/km)</th>
<th>UM/UM$_e$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>173</td>
<td>3.47</td>
<td>100.0</td>
</tr>
<tr>
<td>HullCol ($\eta=0$)</td>
<td>0.00</td>
<td>177</td>
<td>3.55</td>
</tr>
<tr>
<td>HullCol ($\eta=1$)</td>
<td>0.02</td>
<td>176</td>
<td>3.51</td>
</tr>
<tr>
<td>HullCol ($\eta=3$)</td>
<td>0.06</td>
<td>172</td>
<td>3.46</td>
</tr>
<tr>
<td>HullCol ($\eta=5$)</td>
<td>0.11</td>
<td>169</td>
<td>3.43</td>
</tr>
<tr>
<td>HullCol ($\eta=9$)</td>
<td>0.22</td>
<td>161</td>
<td>4.14</td>
</tr>
</tbody>
</table>

![Ultimate moment and curvature versus residual stress normalized by yield stress](image)

Fig. 15 Ultimate moment and curvature versus residual stress normalized by yield stress.

**CONCLUSION**

The results of a test on a very thin box-girder are presented. The load history is composed of 4 cycles of load: two in elastic domain, one up to collapse and beyond, and the last one after collapse. In the first two cycles the same maximum load was applied and it was demonstrated that after the residual stress relief on the first cycle, the behavior of the box became elastic but some structural hysteresis is present.

The collapse of the box girder is due to plating induced failure as result of the high plate’s slenderness. The wave length of plate after collapse is higher than the aspect ratio of the plate confirming that the lowest strength configuration is achieved for such relation in the presence of some degree of lateral edges restraint to inwards movement due to existence of transverse frames.

The post collapse response is smooth for thin boxes since the ultimate strength of the structural components, plating and stiffened plates in compression is low.

The progressive collapse method used for comparison gives very good prediction of the behavior of the structure and its application allows estimating the level of residual stresses presented in the structure as approximately 6% of the yield stress. Also the ultimate curvature is increased as residual stresses increase. The ultimate bending moment degrades 5% for every 10% of residual stresses increase.

Finally the ultimate bending moment of such very thin box girder is well below the first yield bending moment. Thus non-linear analysis including inelastic structural instability is mandatory for analysis of such structures.

**REFERENCES**