INTEGRITY OF OFFSHORE STRUCTURES - 5

Edited by

D. FAULKNER *

M. J. COWLING **

A. INCECIK *

and

P. K. DAS *

**Department of Naval Architecture and Ocean Engineering*

***Department of Mechanical Engineering*

The University of Glasgow, Glasgow, Scotland, U. K.

EMAS

ENGINEERING MATERIALS ADVISORY SERVICES LTD.,
339, Halesowen Road, Cradley Heath, Warley, West Midlands B64 6PH, U.K.

10

APPROXIMATE LOAD SHORTENING CURVES FOR STIFFENED PLATES UNDER UNIAXIAL COMPRESSION

J.M. GORDO and C. GUEDES SOARES

Department of Naval Architecture and Ocean Engineering
(Instituto Superior Técnico), Technical University of Lisbon

ABSTRACT

An approximate method is proposed to describe the load shortening curves of stiffened panels, including the post-buckling behaviour. The model accounts for the influence of plate and stiffener distortions and residual stresses on the ultimate strength of the stiffened plates. The method is compared with the results of a numerical analysis of stiffened plates using a finite element program in the relevant range of plate and column slenderness.

1. INTRODUCTION

The strength of plate elements and stiffened panels has been an important issue in the design of marine structures for a long time. The different viewpoint in the analysis and the design formulations have also been very clear, not only from the point of view of the type of approach adopted but also from the uncertainties involved [1].

While for strength assessment lengthy numerical procedures were adopted, simpler approaches were chosen in the case of design. In the latter, the accuracy and the amount of information provided was traded off by the easiness and speed of application. Several design formulae have been proposed [2-6] to quantify the maximum load carrying capacity of plates and they have been useful for design purposes. However, they only provide information about the maximum load carrying capacity of the plate element, while the numerical formulations [7-11] give information about the full load-shortening behaviour of the plate.

The development of computer based design methods has allowed, in many situations, the strength of structures made of stiffened panels to be evaluated by considering the contribution of each plate element as described by its load-shortening curve. However, even the computer based approaches cannot deal with the numerical strength prediction methods for design purposes. The time required to perform the calculation for each plate element or for each plate-stiffener configuration is still too long to be accepted in normal design work, especially if one considers structures made of many such elements. Therefore, different approaches have been developed to represent the load-shortening curve of stiffened plates based on the load deflection curves of plate elements.

Smith [12] produced average stress–strain curves based on a finite element formulation for the stiffened plate element and a load shortening curve for the strength of each plate element. Plates were grouped by slenderness and three levels of imperfections were considered: slight, average and severe. Linear interpolation between the load-shortening curves in the data-base was used for intermediate slendernesses. Each increasing level of imperfections was associated simultaneously with both increasing residual stresses and distortions. This does not seem to be very appropriate because residual stresses decrease as plate slenderness increases, while distortions increase with slenderness, as discussed for example by Faulkner [2] and Guedes Soares [5].

Billingsley [13] developed his plate model by considering two main contributions to plate effectiveness: the first one, concerned with the edge zones, is based on an effective width concept in which the contribution from the central portion of the plate is the loading supported by an infinitely wide plate. Only the first contribution must be considered in long plates. The model does not take into account either residual stresses and distortions or load shedding after buckling, which is very important for ultimate bending moment estimation.

Adamchak [14] proposed another approach based on the same type of formulation but including also the tripping collapse of the stiffeners.

This brief review shows that different approaches have been developed to represent the load-shortening curves of stiffened plates based on the load deflection

represent the load-shortening curves of stiffened plates based on the load deflection curves of the individual plate elements. However, several of them used a data base of load shortening curves and for given values of plate parameters they interpolated to find the best curve.

The approach taken in this paper is different, in that the load-shortening curves are produced directly from mathematical expressions which have proved to be adequate for design purposes.

2. LOAD-SHORTENING CURVES FOR PLATES UNDER UNI-AXIAL LOADING

The present work considers that the material has an elastic–perfectly plastic behaviour for every average strain. This is not completely correct since it does not consider either the change in tangent modulus beyond the proportional stress nor the hardening after yielding. However, the approximation is quite accurate for structural steel, especially because there is no interest in strains larger than three or four times the yield strain. This behaviour may be represented analytically by:

$$\Phi(\bar{\varepsilon}) \equiv \Phi_e = \begin{cases} -1 & \text{when } \bar{\varepsilon} < -1 \\ \bar{\varepsilon} & \text{when } -1 < \bar{\varepsilon} < 1 \\ 1 & \text{when } \bar{\varepsilon} > 1 \end{cases} \tag{1}$$

where Φ_e is the edge stress ratio, i.e. σ_e / σ_o, where σ_e and σ_o are respectively the edge and yield stress and $\bar{\varepsilon}$ is the average strain ratio, i.e. $\varepsilon_e / \varepsilon_o$, where ε_e is the edge strain and ε_o is the yield strain.

Plate strength under compression depends mainly on its geometry and more precisely on its slenderness. Several approximate expressions have been proposed to represent the collapse strength of plates as a function of its slenderness:

$$\beta_o = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}} \tag{2}$$

which depends on the plate breadth b and thickness t as well as on the material yield stress σ_o and Young modulus E.

It has become traditional to deal with the reduced strength of the plates by equating it to the strength of another plate that has an effective width Φ and collapses at nominal yield stress. Therefore, speaking of effective width or of ultimate strength becomes equivalent.

From the different proposals existing in the literature, the formula proposed by Faulkner has proven to be a good one. According to it, imperfect plates with simply supported edges forced to remain straight under longitudinal loading, have an effective width, Φ_{wo} given by [2]:

$$\Phi_{wo} = \frac{2}{\beta_o} - \frac{1}{\beta_o^2} \quad \text{for } \beta_o > 1 \tag{3a}$$

$$\Phi_{wo} = 1 \quad \text{for } \beta_o \leq 1 \tag{3b}$$

This expression was derived by Faulkner [2] for the case of plates with average initial distortions. Guedes Soares [5] has checked the performance of that formula against more recent experimental data and has extended it to deal explicitly with different levels of initial distortions. He found that the dependence on β incorporated in the Faulkner expression was the correct one to describe the strength of plates without distortions, which were however somewhat stronger. He showed that in this case the adequate expression was:

$$\Phi_o = \frac{2.16}{\beta_o} - \frac{1.08}{\beta_o^2} \quad \text{for } \beta_o > 1 \tag{4}$$

This expression leads to values higher than 1.0 for stocky plates which has been observed in different experiments and can be attributed to the strength-ening effect of the edges being forced to remain straight by the adjacent stiffeners. The expressions (3) and (4) represent the ultimate collapse strength of plates, a situation in which the edge stresses are equal to yield. It is assumed here that the load shortening curve of the plate can be represented by the same expression where now the slenderness β, is defined for each strain by:

$$\beta = \frac{b}{t} \cdot \sqrt{\varepsilon} \tag{5}$$

which is related to the nominal slenderness, β_o, by:

$$\beta = \beta_o \cdot \sqrt{\varepsilon}\,, \tag{6}$$

Therefore, the effective width at a given strain is obtained, by substituting β_o by β in eqn. (3):

$$\Phi_w = \frac{2}{\beta} - \frac{1}{\beta^2} \quad \text{for } \beta > 1 \tag{7}$$

Thus, for a given plate, the effective width is changing from a value close to 1 to lower values as the loading is increasing. No substantial reduction in effective width takes place when buckling occurs $(\beta \equiv \beta_o)$ and the discontinuity of the tangent modulus at the buckling point is only due to the yielding of edge strips.

The normalised average stress of the plate is given by the product of the edge stress (eqn.1) and the effective width corresponding to this stress level (eqn. 7), as indicated in Fig. 1:

$$\Phi_a = \Phi_e \cdot \Phi_w \tag{8}$$

As can be seen in Fig.1, the stress–strain curves have their maximum value at yield strain, if residual stresses are not considered. Figure 2 shows average stress–strain curves for several levels of slenderness. The model adopted has a *decreasing* loading capacity after buckling, if it is considered that there is an increased reduction on effective width after yielding of the plate's edges.

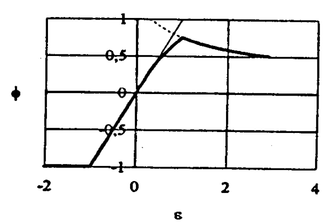

Fig. 1 - Construction of a stress-strain curve for a plate element $\beta_o = 1.6$.

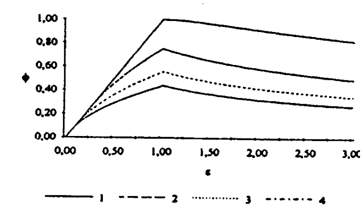

Fig. 2 - Stress-strain curves for plates of $\beta_o = 1$, 2, 3 and 4.

2.1. Effect of Residual Stresses

The effect of residual stresses on the strength of plates has been the subject of several studies. Some of them attempted to determine the effect of the level of residual stresses on the ultimate strength [1,5] incorporating an explicit formula for that variation in the strength prediction equation. Others were more interested in the influence of residual stresses in the whole stress–strain curve [7] and, in that case, no expression is given to compute the variation of the strength.

The approach proposed by Crisfield [7] takes explicitly into consideration the effect of the tension strips near the edges. Crisfield considered separately the regions of the plate under compression and tension. For this last one he considered a linear elastic behaviour from $\bar{\varepsilon} = 0$ to 2 and beyond this strain the strip yields in compression. In the central zone, initially under compression, he considered that the behaviour is similar to the one of an elastic plate without residual stresses and with an initial strain of $\bar{\varepsilon}_r$. So this zone reaches its maximum loading capacity at a strain of $1-\bar{\varepsilon}_r$.

This effect on an elastic perfectly plastic plate produces an apparent variation on the tangent modulus at a strain of $1-\bar{\varepsilon}_r$ mainly due to the yielding of the central region of the plate. Beyond that strain the only contribution to the increase of strength is from the edges strips. These strips can be loaded until $2\varepsilon_o$ from where the plate behaves plastically, as shown in Fig. 3. Analytically the straight line between the elastic region and the yield stress is given by:

$$\Phi_r = \frac{\bar{\sigma}_r \cdot \bar{\varepsilon} + 1 - \bar{\sigma}_r}{1 + \bar{\sigma}_r} \qquad (9)$$

In the present formulation it is considered that the plate with residual stresses has a strength given by:

$$\Phi_{rp} = \Phi_w \cdot \Phi_r \qquad (10)$$

where Φ_w is eqn (7) and Φ_r is the representation of the elasto–plastic behaviour of the plate corrected by the existence of residual stresses, as shown in Fig. 4. This function may be written for the whole range of strains as:

$$\Phi_r = \max\left\{-1, \min\left[1, \bar{\varepsilon}, \frac{\bar{\sigma}_r \cdot \bar{\varepsilon} + 1 - \bar{\sigma}_r}{1 + \bar{\sigma}_r}\right]\right\} \qquad (11)$$

This expression represents a rough approximation to the behaviour of plates in tension, $\bar{\varepsilon} < 0$, because there is not a perfect identification between the model with residual stresses and the function Φ_r, with respect to the tangent modulus.

In fact, if one analyses the model, it can easily be shown that the structural tangent modulus must be $E_t = (b - 2\eta t)E / b$ when $-1 < \bar{\varepsilon} < 0$, but at Φ_r (eqn 11) the tangent modulus is $E_t = E$. This solution is considered to be better since it ensures the continuity of tangent modulus at $\bar{\varepsilon} = 0$ and it seems to better represent the behaviour of the real plate. Note that η is the weld tension block parameter [2].

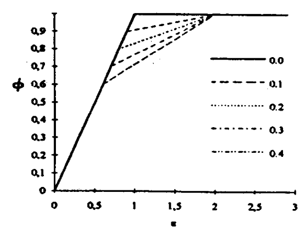

Fig. 3 - Effect on material behaviour due to residual stresses up to 0.4 σ_o.

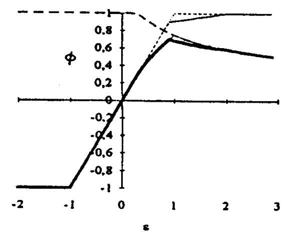

Fig.4 - Construction of a stress-strain curve for a plate with residual stresses.

The practical consequence of the use of one or other interpretation is exclusively on the flexibility of the overall section and so one obtains a higher value for the effective section modulus with the model adopted.

Two important points must be highlighted about the adopted model:

- Φ_w is not affected by the level of residual stresses, which is the same as saying that the eventual loss of effectiveness of the plate caused by the presence of residual stresses is exclusively due to the plastic behaviour of the material.
- the imposition of the continuity in the tangent modulus at $\bar{\varepsilon} = 0$ is an attempt to reproduce correctly the transition between tension and compression. However, the model might be too rigid for strains near yielding in tension. More realistic solutions may be achieved by considering sophisticated patterns for residual stress distributions, such as parabolic curves on the tension strips.

The strain at which the plate reaches its collapse load decreases with increasing level of residual stresses until a point at which it jumps to a strain of $2\varepsilon_o$. The sudden change on ultimate strain results directly from the adopted model for residual stresses distribution, and if a smoother transition between tension and compression fields is considered then a smoother change is obtained and the curves approach the ones recommended by Smith [12].

Figure 5 indicates the load shortening curves for different levels of residual stresses for a plate with a slenderness of 2 showing that, for large strains ($\bar{\varepsilon} > 2$) the curves coincide. Figure 6 shows comparisons between two approximate curves of different levels of residual stresses and the calculations performed with a finite element code [11]. The differences on the maximum load carrying capacity are very small but the approximate curves are higher in the post-collapse range. However, the shedding pattern obtained with this FEM code is quite sensitive to the aspect ratio of the plate. For this particular case a plate of an aspect ratio of 3 was chosen, but for plates of lower aspect ratio a better agreement is expected.

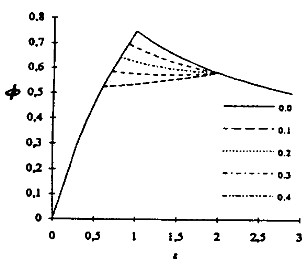

Fig. 5 - Behaviour of a plate of β=2 with different levels of residual stresses.

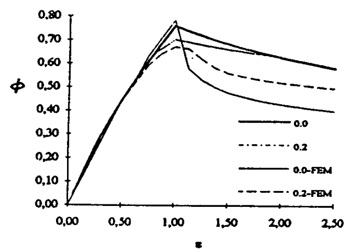

Fig. 6 - Comparison between approximate method and FEM calculations for a plate element with β=2 and *a/b*=3.

2.2. Effect of Initial Deformations

The plate elements of marine structures show distortions that result from the fabrication in steelworks, from impacts during transportation, from the manufacturing process in shipyards, and from operation. The first two kinds are corrected in the shipyard, but the third one remains for the entire life of the structure and increases during service until replacement of the plating occurs.

In *tension*, the dominant effect of the distortions is the variation of the initial tangent modulus, which has a lower value than the Young's modulus. As a consequence, the plates in tension will have a lower rigidity than the perfect plate. However, for the usual level of distortions, the consequences are irrelevant and may be ignored.

In *compression*, out–of–plane deformations are much more important. Their presence in plate elements makes the load–end shortening curves smoother

near collapse, so there is not then a critical collapse load. In other words, the sudden collapse that characterises the almost perfect plates, as shown in Fig. 6, disappears and the behaviour of the plates is smoother as the distortions are greater.

Guedes Soares [3] has quantified the loss of strength of imperfect plates due to initial imperfections or residual stresses or both simultaneously, which is the most common situation. The strength of a plate with initial distortions is given by:

$$\Phi_s = \Phi_o \Phi_\delta \tag{12}$$

where Φ_o is given by eqn. (4) and the strength degradation due to initial distortions is given by:

$$\Phi_\delta = 1 - (0.626 - 0.121\beta)\bar{\delta} \tag{13}$$

when the initial distortions coexist with residual stresses, an interaction factor:

$$\Phi_{R\delta} = 0.76 + 0.01\eta + 0.24\bar{\delta} + 0.1\beta \tag{14}$$

must be introduced, leading to the prediction formula for the collapse load of imperfect plates [3]:

$$\Phi_u = \Phi_o . \Phi_\delta . \Phi_R . \Phi_{R\delta} \tag{15}$$

where Φ_R is the effect of residual stresses.

3. COLLAPSE STRENGTH OF STIFFENED PLATES

In plate panels, the longitudinal stiffeners have the main function of providing the necessary support to the plates ensuring that they retain the required strength. To fulfil this function, stiffeners must have adequate rigidity, and the spacing between them must be chosen according to the main characteristics of the plate, namely, its thickness and yield stress. The slenderness of the plate has to be designed in such a way that the ultimate average stress is kept closer to the yield stress as much as possible.

The analysis of stiffened plates was performed by several researchers and many solutions to the problem were presented over the years. The prediction of the panel behaviour has led to the development of several techniques such as non–linear finite element methods [7-11] or more simplified formulations applying the beam–column concept [12-14]. Common to all of them is the need of the application of an incremental end–shortening if a realistic description of the post–buckling behaviour is required. Also common to the latter formulation is the use of load–end shortening curves for simply supported plates carried out on separate studies which are able to describe the loss of plate stiffness after buckling. Methods involving the solution of the equilibrium equation were developed by Little [15], Moolani[16] and Crisfield [7].

Design methods to determine the ultimate load of the panels were presented among others by Faulkner et al [17] based on the Johnson–Ostenfeld column approach, by Carlsen [18] and Dwight and Little [19] based on the Perry-Robertson formulation.

Failure of panels is usually classified as: plate induced failure, column–like failure, tripping of stiffeners and overall grillage failure. This last one is normally avoided by ensuring that transverse frames are of adequate size therefore it is not considered in this study. The first one occurs when the stiffener is sufficiently stocky and the plate has a critical elastic stress lower than yield stress. The second failure mode is mainly due to excessive slenderness of the column (stiffener and effective associated plate acting together) and failure may be towards the plate or towards the stiffener, depending on the column's initial shape and the type of loading considered, i.e., eccentrically applied or not, following the shift of the neutral axis or not. In a continuous panel it is usual that the failure is towards the plate in one span and towards the stiffener in the adjacent span. The third mode of failure is the consequence of a lack of torsional rigidity of the stiffener. Interaction with the plate buckling mode may also occur inducing premature tripping.

Sometimes the first and second modes are incorporated in the same group because the buckled shape of the panel is similar and is normally towards the stiffener.

3.1 Plate Induced Failure

In this mode of failure the stiffener is able to sustain stresses near yield but the plate can only sustain its ultimate stress, σ_u. Thus the ultimate strength of the column is:

$$\Phi_{uc} = \frac{A_s + b_e t}{A_s + bt} \tag{16}$$

where A_s is the stiffener area, b and b_e are respectively the width and the effective width of the plate, and t is the thickness of the plate.

To obtain the average load-end shortening curves of the column it is assumed that the stiffener has an elastic-perfectly plastic behaviour, eqn (1), and that the plate behaves according to eqn (7) and so eqn(16) is changed to:

$$\Phi_{ac} = \Phi_e \cdot \frac{A_s + \Phi_w(\bar{\varepsilon}) \cdot bt}{A_s + bt} \tag{17}$$

where Φ_e is the edge stress and $\Phi_w(\bar{\varepsilon})$ is the effective width of the plate for each strain level (eqn 7).

3.2 Flexural Buckling of Columns

Based on the Johnson-Ostenfeld formulation, which accounts for inelastic effects of column's buckling, Faulkner et al [17] proposed a model for the strength of thin stiffened plates where it is considered that both the stiffener and an effective strip of the associated plate are subjected to an edge stress, σ_e. The maximum edge stress that this column can sustain is related to the yield stress by the Johnson-Ostenfeld approach, but the model used to calculate the flexural buckling rigidity of the column must consider a tangent effective width of the associated plate in order to include the reduction of the tangent modulus in a bending situation. Under this approach the ultimate strength of the column is given by:

$$\Phi_{uc} = \frac{\sigma_{ue}}{\sigma_o} \cdot \left\{ \frac{A_s + b_e t}{A_s + bt} \right\} \tag{18}$$

and the maximum edge stress ratio is related to the column and material properties by:

$$\frac{\sigma_{ue}}{\sigma_o} = 1 - \frac{1}{4} \cdot \frac{\sigma_o}{\sigma_E} \quad for\ \sigma_E \geq 0.5\sigma_o \tag{19a}$$

$$\frac{\sigma_{ue}}{\sigma_o} = \frac{\sigma_E}{\sigma_o} \quad for\ \sigma_E \leq 0.5\sigma_o \tag{19b}$$

where σ_E is the Euler stress defined as being equal to $\sigma_E = (\pi r_{\alpha} / a)^2 E$. The equivalent moment of inertia is defined by the relation:

$$I_{ce} = r_{ce}^2 \cdot (A_s + b_e t) \tag{20}$$

and the tangent effective width which must be used in (20) to calculate the reduced moment of inertia was derived by Faulkner [2] as:

$$\frac{b_e'}{b} = \frac{1}{\beta} \cdot \sqrt{\frac{\sigma_o}{\sigma_e}} \tag{21}$$

Guedes Soares and Soreide [20] have shown that this formulation predicts the mean strength of stiffened plates accurately, when comparing it to a set of available experiments.

The above relation might be changed and generalised to every strain as in last section by considering that the Euler stress ratio, Φ_E, has an instantaneous value as a consequence of the actual strain. So, the above equations might be redefined, and one obtains for the nominal Euler stress ratio:

$$\Phi_E = \left(\frac{\pi}{\lambda}\right)^2 \tag{22}$$

where the column slenderness is $\lambda = (a / r_{ce})\sqrt{\varepsilon_o}$

and replacing ε_o by ε the Euler stress ratio at a strain ε is:

$$\Phi_E(\bar{\varepsilon}) = \frac{\Phi_E}{\varepsilon} \tag{23}$$

where the column slenderness λ is varying with the strain level due to the variation of effective widths. Modifying eqn (19a,b) to extend it to every strain level, and assuming that the relation is still valid, one obtains the Johnson-Ostenfeld contribution which corresponds to the first term of the second member of eqn (19) but now for any strain:

$$\Phi_{jo}(\bar{\varepsilon}) = \left(1 - \frac{1}{4 \cdot \Phi_E(\bar{\varepsilon})}\right) \cdot \Phi_e \qquad \text{for } \Phi_E(\bar{\varepsilon}) > 0.5 \tag{24a}$$

$$\Phi_{jo}(\bar{\varepsilon}) = \Phi_E(\bar{\varepsilon}) \cdot \Phi_e \qquad \text{for } \Phi_E(\bar{\varepsilon}) < 0.5 \tag{24b}$$

and finally the expression to calculate the load-end shortening curves, including buckling inelastic effects, might be written as:

$$\Phi_{ab}(\bar{\varepsilon}) = \Phi_{jo}(\bar{\varepsilon}) \cdot \frac{A_s + \Phi_w(\bar{\varepsilon}) \cdot bt}{A_s + bt} \tag{25}$$

where $\Phi_{ab}(\bar{\varepsilon})$ is the average stress of a column composed by a stiffener of area A_s and a plate of area bt under a strain $\bar{\varepsilon}$.

As to the last remark, it must be said that this approach is only valid assuming that initial deformations are small and that load diffusion ensures that no eccentricity of the load is present, which seems to be the case. Allowance for residual stresses is automatically taken into account from the application of corrective formula to plate strength already developed, eqn (11). Improvements are still possible if the residual stresses of the stiffener are considered.

3.3 Tripping of Stiffeners

This mode of panel's failure is one of the most dangerous ones because it is always associated with a very quick shed of load carrying capacity of the column. Lateral-torsional instability may occur alone by twisting of the stiffener about its line of attachment to the plating, developing a partial or full hinge at the intersection, or induced by flexural buckling especially if the deflected shape of the column is towards the plate. In that case, the stiffener will be subjected to a higher stress than the average column stress and the critical tripping stress could be easily reached, followed by a deep load shedding.

Tripping involves a rotation of the stiffener about a hinge which is usually considered to be located on the connection of the stiffener to the plating, and vertical flexure in the principal direction of the stiffener.

3.3.1 Elastic tripping stress: There are not many studies about tripping and the present work will follow the approach presented by Faulkner [17,21] and by Adamchak[22], using them to determine the tripping stress and to estimate a pattern for load shedding after tripping. The approach is based on the Rayleigh's principle to obtain the elastic critical stress for tripping. Corrections to the elastic tripping stresses are presented which are intended to incorporate non-linear behaviour of both plate and stiffener.

The proposed approach balances the torsional, sideways bending, warping, and rotational spring strain energies with the elastic tripping strain energy:

$$I_p \sigma_T = GJ + \frac{m^2 \pi^2 E T_p}{a^2} + \frac{C_s a^2}{m^2 \pi^2} \tag{26}$$

where J is the St. Venant torsional constant, T_p is an appropriate tripping parameter that includes both sideways bending $(I_z . \bar{z}^2)$ and longitudinal warping $(\Gamma \cong I_z d^2)$ contributions and is defined as:

$$T_p = I_z \bar{z}^{-2} + \Gamma \tag{27}$$

where $\bar{z}$ is the distance of the stiffener centroid from the toe, and C_s is the elastic rotational spring stiffness per unit length of the toe which can be derived as:

$$C_s = \frac{Et^3}{2.73b}\left[1+\left(\frac{m b}{a}\right)^2\right]^2 \tag{28}$$

Equation (26) is applicable when considering a constant constraint along the toe, but a more accurate approach can be obtained if one accounts for the destabilising moments induced by the plate's loaded shape.

Faulkner proposed that the rotational constraint might be approximated by a linear interaction based on the analysis of dynamic behaviour of ship grillages [23]:

$$\frac{C}{C_s} + \frac{\sigma}{\sigma_{cr}} = 1 \quad \text{for } \frac{\sigma}{\sigma_{cr}} \le 2 \tag{29}$$

where σ_{cr} is the elastic critical stress of a simply supported plate:

$$\sigma_{cr} = \frac{\pi^2}{12\left(1-\nu^2\right)} E\left(\frac{t}{b}\right)^2 \left[\frac{m_o b}{a} + \frac{a}{m_o b}\right]^2 \tag{30}$$

and m_o is an integer representing the mode of possible collapse of the plate element. Introducing eqn (30) into eqn (26) the tripping stress might be determined:

$$\sigma_T = \left(GJ + \frac{m^2\pi^2 ET_p}{a^2} + \frac{C_s a^2}{m^2\pi^2}\right) \Big/ \left(I_p + \frac{C_s a^2}{m^2\pi^2\sigma_{cr}} k\right) \tag{31}$$

The coefficient k is an attempt to include the interaction between the plate and the stiffener and Faulkner recommended the values 1.0, 0.0, 0.5 and 0.0 respectively for m_o/m = 1, 2, 3 and more than 3.

Theoretically two remarks must be made on this approach. Firstly, the third value must be 0.33 because only one third of the plate is destabilising the stiffener. On the other hand, if the plate is destabilising the stiffener then the stiffener is stabilising the plate and some degree of rotational constraint is applied to the plate. Therefore the assumed simply supported plate's boundary conditions

might seem to be a little conservative. Furthermore, any consideration about the plastification of the toe is not taken into account, which might be very important from the point of view of load-end shortening curves of columns since, if some plastification occurs, then the toe's spring constant will be reduced.

3.3.2 Inelastic effects: To estimate the inelastic effects, Faulkner [21] recommended the use of the $\sqrt{E_t E}$ instead of the E_t to be used on column flexural buckling, due to local bending effects, and a tangent modulus defined by the Ostenfeld-Bleich quadratic parabola:

$$\frac{E_t}{E} = \frac{\Phi\cdot(1-\Phi)}{p_r\cdot(1-p_r)} \tag{32}$$

where p_r is the proportional limit stress ratio.

The inelastic tripping stress of the stiffener with effective associated plate will become:

$$\Phi_{Ti} = \Phi_T^2 \Big/ \left[\Phi_T^2 + p_r\cdot(1-p_r)\right] \quad \text{if } \Phi_T > p_r \tag{33a}$$

$$\Phi_{Ti} = \Phi_T \quad \text{if } \Phi_T < p_r \tag{33b}$$

where p_r is recommended to be taken as 0.8 because of the presence of largely tensile stresses due to residual stresses.

The ultimate average tripping stress of the panel will be finally:

$$\Phi_{Tu} = \Phi_{Ti}\cdot\frac{A_s + b_e t}{A_s + bt} \tag{34}$$

where the effective width b_e must be calculated for a slenderness $\beta = \beta_o\cdot\sqrt{\varepsilon_{Ti}}$ using equation (3). The approach used to determine b_e implicitly considers that the stiffener behaves elastically until σ_{Ti} is reached and so naturally the corresponding strain will be $\varepsilon_{Ti} = \sigma_{Ti}/E$, which is also the plate's average shortening.

3.3.3 Behaviour pre and post tripping: The equations of last section are only used by Faulkner and Adamchak to predict the ultimate tripping stress. Thus, the above equations were modified in this work according to the same principles of section 3.2 but now it is only necessary to predict the behaviour of the column after tripping (say for $\varepsilon \geq \varepsilon_{Ti}$) because before tripping the column's behaviour will be governed by flexural bending. Considering that the stiffener has a perfectly elastic behaviour for tripping stress calculations then equations (24a,b) for flexural behaviour will always be lower than the elastic one (the effective width is the same in both analyses) and thus flexural derived average curves will govern the pre-buckling behaviour.

The criteria followed considers the minimum tripping stress, eqn (30), and if this one is lower than the yield stress, it is multiplied by the effective area ratio:

$$\Phi_T(\bar{\varepsilon}) = \Phi_{T\min} \cdot \frac{A_s + \Phi_w(\bar{\varepsilon}) \cdot bt}{A_s + bt} \tag{35}$$

which does not account for column's load shedding after tripping but only to the loss of the plate's effectiveness.

To approach the shedding of load after tripping one very simple formula is used as a result of a qualitative analysis of the available average load-end shortening curves using finite element methods, which consists of the reduction of the column's carrying capacity by the ratio of the tripping strain and the actual strain:

$$\Phi_{Tc}(\bar{\varepsilon}) = \Phi_T(\bar{\varepsilon}) \cdot \frac{\varepsilon_T}{\varepsilon} \tag{36}$$

This is a rough approximation to the problem but it seems to be not far away from the reality and the results obtained with it are very consistent.

4. LOAD-END SHORTENING CURVES OF BEAM-COLUMNS

The model used to describe the behaviour of columns is based on the equations already derived in previous sections. Each equation is related to one particular mode of failure and at every end-shortening of the column the average stress to be considered is always the lowest one among the calculated stresses for each mode of failure:

- plate failure using Φ_{uc} from equation (17),
- flexural column failure using Φ_{ab} from equation (18)
- tripping failure of stiffeners using equation (35) or (36).

It is evident that the stresses predicted by Φ_{uc} are always higher than those predicted by Φ_{ab} but in spite of this they are considered to allow comparisons

between models. These comparisons may be very helpful in the design of panels providing qualitative information about the 'best' geometry to be chosen.

Three kinds of distortions co-exist in a stiffened panel: initial imperfections of the plate element, out-of plane of the stiffener and lack of perpendicularity between stiffener and plate along the span. The first one decreases the ultimate strength of the plate and consequently decreases the strength of the panel. Its effect on plate behaviour has already been discussed in section 2.2.

Out-of-plane of the stiffener induces bending stresses along the span of the stiffener increasing the local stress in extreme fibres and so the actual axial carrying capacity of the column will be lower than that with a straighter stiffener because yield stress will be reached at lower axial average stress. These distortions result from the manufacturing process, i.e. mounting defects and especially as a consequence of welding.

The remainder is mainly a consequence of mounting deficiencies in earlier stages of the construction or of the use of alternate and intermittent welding on light stiffeners. It is especially important in torsional behaviour of the column by reducing the tripping stress.

4.1 Comparisons Between the Approximate Method and the FEM

The prediction of the approximate method proposed here was compared with the numerical predictions of the computer program PANFEM, developed by Jazukiewicz et al [11]. This is a non-linear finite element code for flat plates and flat stiffened panels with imperfections (initial deflections and residual stresses) under lateral and in-plane loads. Large displacement effects are handled using a total Lagrangian formulation. Isotropic hardening of material is assumed and the Huber-von Mises yield criterion, together with the associated Prandl-Reuss flow rule, are applied. The Newton-Raphson and the Modified Newton-Raphson methods are utilised, combined with the Powell and Simons or the Crisfield arc length procedure to handle both pre- and post-collapse behaviour of plates and stiffened plates. The program predictions were compared with well documented experimental results and showed a very good correlation [24]. It has already been used to simulate the strength of plate elements with different levels of imperfections [25].

Two models taken from existing panels [26] were compared, one with a bar as stiffener, and the other was a T stiffened plate. For the first, the approximate method predicts the collapse at a strength factor of 0.770 and the corresponding strain was 85% of the yield strain, while the FEM predicts a strength ratio of 0.759 at a strain of 0.79 of the yield strain. The difference between both methods is of the order of 1.5% in the strength prediction and of 6% in the corresponding strain and the agreement over the whole curve is significant, as shown in Fig. 6. The T type section also gives good results and confirms that the approximate method is quite accurate.

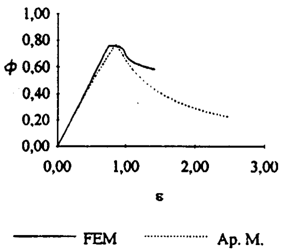

Fig. 6 - Average stress-strain curves of a column element of a bar stiffener with β=1.56 and λ=1.36

To test the method over a wide range of plate and column slenderness the strength of a series of stiffened plates were calculated both with the proposed approximate method and the finite element program.

The models were T sections with associated plating. The geometry of the panels was conventional and the thicknesses of plate, web and flange were chosen to be approximately equal at the middle of the plate and column slenderness ranges. Four levels of plate slenderness (β = 1.16, 2.32, 3.09 and 4.63) and three levels of column slenderness (λ = 0.6, 1.2 and 2.5) were considered to cover a large range of these two parameters. Two additional plates with intermediate column slenderness were considered to cover closely the range of λ which, it should be noted, includes all plating. This column slenderness is π times that usually used $\left(\lambda = \sqrt{\sigma_o / \sigma_E}\right)$.

The agreement on the ultimate strength between the approximate method and FEM calculations is quite good for the most part of the plates, as shown in Table 1, where the collapse strength predicted by the approximate method is shown when tripping failure is accounted for (ultimate strength) or not (flexural strength). The best results are obtained when both methods predict collapse by flexural buckling, where the difference does not exceed 5%. The relatively high difference (12%) at low β and high λ (specimen 12), may be due to the fact that the FEM models only allows for intial plate distortions but assumes the web and flange of the stiffeners are initially flat and that the line of toe connection of the web to the plate is initially straight. This provides greater rigidity than in practical structures. It could be the reason why FEM results show a larger secant modulus than the approximate method in the pre-collapse region for all the tested models.

INTEGRITY OF OFFSHORE STRUCTURES – 5

Table 1: Ultimate strength obtained by the Approximate Method (with tripping formulation or only with flexural buckling) and the FEM

Name	β	λ	Ult. St.	Flex. St.	FEM	FEM/U.S.	FEM/F.S.
10	1,16	0,65	0,98	0,98	1,00	1,02	1,02
11	1,16	1,30	0,94	0,94	0,99	1,05	1,05
12	1,16	2,60	0,82	0,82	0,92	1,12	1,12
20	2,32	0,60	0,80	0,80	0,81	1,01	1,01
21	2,32	1,20	0,78	0,78	0,79	1,01	1,01
21A	2,32	1,55	0,76	0,76	0,77	1,01	1,01
21B	2,32	1,91	0,74	0,74	0,75	1,02	1,02
22	2,32	2,39	0,69	0,69	0,73	1,05	1,05
30	3,09	0,61	0,76	0,76	0,79	1,04	1,04
31	3,09	1,21	0,74	0,74	0,73	0,99	0,99
32	3,09	2,42	0,65	0,65	0,66	1,01	1,01
40	4,63	0,64	0,64	0,74	0,76	1,19	1,03
41	4,63	1,27	0,60	0,71	0,71	1,17	0,99
42	4,63	2,55	0,56	0,62	0,64	1,14	1,03

Figures 7 to 10 plot the load-end shortening curves of four stiffened plates with both methods. While in Figs 6, 8 and 9 the agreement over the range of strains is good, Fig. 10 shows one particular panel where the approximate method predicts tripping and the FE method does not.

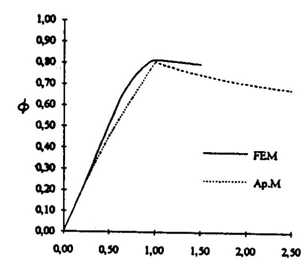

Fig. 7 - Average stress-strain curves of a column with β=2.32 and λ=0.60

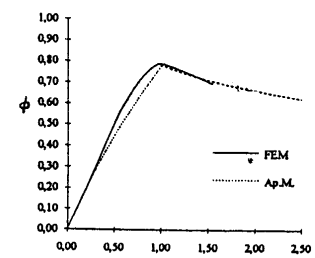

Fig. 8 - Average stress-strain curves of a column with β=2.32 and λ=1.20

The group of stiffened plates with high β (series 40) shows different behaviour depending on the method. While the approximate method predicts tripping collapse for the three panels, the FEM produces load-end shortening curves similar to those predicted by the approximate method when tripping is not considered, Fig. 10. Again in these cases the lack of stiffener imperfections may be responsible for the difference in the results.

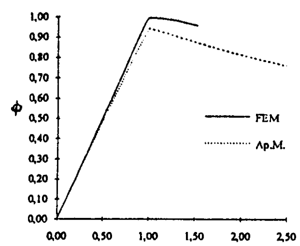

Fig. 9 - Average stress-strain curves of a column with β=1.16 and λ=1.30

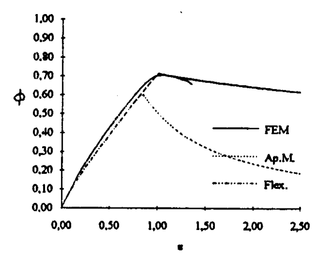

Fig. 10 - Average stress-strain curves of a column with β=4.63 and λ=1.27

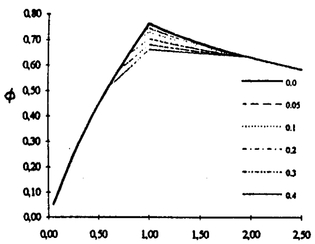

Fig. 11 - Effect of residual stresses on average stress-strain curves of a column element with β =2.3 and λ=1.5 for η = 0 to 0.4

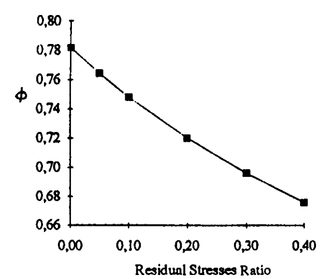

Fig. 12 - Decrease on ultimate strength due to residual stresses in specimen 21.

The impact of residual stresses on the average stress-strain curves is shown in Fig. 11. It corresponds to a decrease of the ultimate strength which may be very high if the plates are heavily welded and it can represent a decrease of 10% on strength, Fig. 12. In the curves of Fig. 11 there are two points in the region of

compression where the slope of the curve has a discontinuity: one is the ultimat stress and the other is the initiation of yielding on the central part of the plate whicl is subjected to compressive residual stresses.

To complete the comparison between the approximate method and the finit element results, an account must be given of the computation time. The approximat method takes about 15 seconds of run time in a 386 personal computer, while th finite element program takes around 10 hours in a 486 personal computer with larg RAM memory. In a 486 without a large RAM the time will be 4 to 5 times longer It was not tried on a 386 computer but it is estimated that the time would be severa days.

5. CONCLUSIONS

The present approximate method for predicting the load-shortening curve of columns under predominantly uni-axial loads seems to represent quite well the behaviour of the stiffened panels both in the ultimate strength prediction as well a in the pre- and post-buckling behaviour, especially when the column buckles by flexural collapse.

The main advantage of the approximate method relative to the finite elemen method results from the time consumption both in the creation of the model and in the CPU time, which ranges from a few seconds in one method to 10 hours typically in the other. On the other hand, with a finite element program one must be very careful about the imposed boundary conditions and the plate and stiffener imperfections if a realistic load-end shortening curve is desired. With the proposed approximate method average distortions are automatically accounted for, while residual stresses are easily incorporated.

Finally, it seems to be necessary to have a better understanding of the tripping phenomena and collapse load.

6. ACKNOWLEGDEMENTS

Part of this work was taken from the thesis of the first author, part of which was performed at Glasgow University. The authors are indebted to Professors D. Faulkner and M. Kmiecik for fruitful discussions on the subject, and to Mr. José Manuel Cruz for his colaboration in performing the FEM calculations.

REFERENCES

1. GUEDES SOARES, C.: 'Uncertainty modelling in plate buckling', structural safety, vol. 5, pp. 17-34, 1988.

2. FAULKNER, D.: 'A review of effective plating for use in the analysis of stiffened plating in bending and compression', Journal of Ship Research, vol. 19, pp. 1-17, 1975.

3. SOREIDE, T.H. and CZUJKO, J.: 'Load-carrying capacity of plates under combined lateral load and axial/biaxial compression', Proc. of 2nd Intl Symposium on Practical Design in Shipbuilding (PRADS), Tokyo, 1983.

4. UEDA, Y. and YAO, T.: 'The influence of complex initial deflection modes on the behaviour and ultimate strength of rectangular plates in compression', J.Construct. Steel Research, vol. 5, pp. 265-302, 1985.

5. GUEDES SOARES, C.: 'Design equation for the compressive strength of unstiffened plate elements with initial imperfections', J.Construct. Steel Research, vol. 9, pp. 287–310, 1988.

6. GUEDES SOARES, C.: 'Design equation for ship plate elements under uniaxial compression', J.Cons. Steel Research, vol. 22, pp. 99-114, 1992.

7. CRISFIELD, M.A.: 'Full range analysis of steel plates and stiffened plating under uniaxial compression', Proc. Inst. Civil Engrs., vol. 59, 1975.

8. FRIEZE, P.A., DOWLING, P.J. and HOBBS, R.E.: 'Ultimate load behaviour of plates in compression', Steel Plated Structures, Crosby Lockwood Staples, London, pp 24–50, 1977.

9. HARDING, J.E., HOBBS, R.E. and NEAL, B.G.: 'The elasto-plastic analysis of imperfect square plates under in-plane loading', Proc. Inst. Civil Engrs, 63, 1977.

10. LITTLE, G.H.: 'Rapid analysis of plate collapse by like-energy minimisation', J.Mech. Sciences, 19, 1977.

11. JAZUKIEWICZ A., KMIECIK M., TACZALA M. and MAJKA K.: 'System of programs for IBM PC/AT for nonlinear analysis of plates and panels by Finite Element Method', Budowinctwo i Gospodarka Morska, nr. 11-12, 1990 (in Polish).

12. SMITH, C.S.: 'Influence of local compressive failure on ultimate longitudinal strength of a ship's hull', Proc. Intl Symposium on Practical Design in Shipbuilding, (PRADS), Tokyo, pp 73-79, 1977.

13. BILLINGSLEY, D.W.: 'Hull girder response to extreme bending moments', Proc. 5th STAR Symposium SNAME, pp. 51-64, 1980.

14. ADAMCHACK, J. C.: 'Approximate method for estimating the collapse of a ship's hull in preliminary design", Proc. Ship Structure Symposium '84, SNAME, pp. 37-61, 1980.

15. LITTLE, G.H.: 'Stiffened steel compression panels–theoretical failure analysis', The Structural Engineer, 54, n.12, 1976.

16. MOOLANI, F.M. and DOWLING, J.P.: 'Ultimate load behaviour of stiffened plates in compression', Steel Plated Structures, Crosby Lockwood Staples, London, pp. 51–88, 1977.

17. FAULKNER, D., ADAMCHACK, J.C., SNIDER, M and VETTER, M.F.: 'Synthesis of welded grillages to withstand compression and normal loads', Computers and Structures, vol. 3, pp. 221-246, 1973.

18. CARLSEN, C.A.: 'Simplified collapse analysis of stiffened plates', Norwegian Maritime Research, vol. 4, 1977.

19. DWIGHT, J.B. and LITTLE, G.H.: 'Stiffened steel compression flanges - a simpler approach', The Structural Engineer, vol. 54A, pp.501-509, 1976.

20. GUEDES SOARES, C. and SOREIDE, T.H.: 'Behaviour and design of stiffened plates under predominantly compressive loads', Intl Shipbuilding Progress, vol. 30, no.341, pp. 13–27, 1987.

21. FAULKNER, D.: 'Toward a better understanding of compression induced tripping', Steel and Aluminium Structures, 3, R. Narayanan (Ed.), Elsevier Applied Science, pp.159–175, 1987.

22. ADAMCHAK, J.C.: 'Design equations for tripping of stiffeners under in-plane and lateral loads', DTNSRDC Report 79/064, Bethesda, Maryland, Oct .1979.

23. SMITH, C.S. and FAULKNER, D.: 'Dynamic behaviour of partially constrained ship grillages', The Shock and Vibration Bulletin, 40, part 7, Naval Research Laboratory, Washington DC, 1969.

24. KMIECIK, M.: 'The influence of imperfections on the load carrying capacity of plates under uniaxial compression', Ship Technology Research, vol. 39, pp. 17-27, 1992.

25. GUEDES SOARES, C. and KMIECIK, M.: 'Simulation of the ultimate compressive strength of unstiffened rectangular plates', Proc. Charles Smith Memorial Conference, DRA-ARE, Dunfermline, 13,14 July 1992.

26. RUTHERFORD, S.E. AND CALDWELL, J.B.: 'Ultimate longitudinal strength of ships: a case study', SNAME Annual Meeting, no. 14, 1990.

27. GUEDES SOARES, C. and FAULKNER, D.: 'Probabilistic modelling of the effect of initial imperfections on the compressive strength of rectangular plates', Proc. 3rd Intl Symposium on Practical Design of Ships and Mobile Units (PRADS), Trondheim, 2, pp.783–795, 1987.